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Fixed point theorems for mappings with convex diminishing diameters on cone metric spaces

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ABSTRACT

In this work, Cantor's intersection theorem is extended to cone metric spaces and as an application, a fixed point theorem is derived for mappings with locally power diminishing diameters.

1. Introduction and preliminaries

Cone metric spaces were rediscovered by Huang and Zhang [1] who replaced the set of real numbers by an ordered Banach space in the definition of the metric, and obtained some fixed point theorems for contractive type mappings. Although this notion was introduced in the middle of the 20th century, Huang and Zhang [1] defined the convergence via interior points of the cone P, by which the order in Banach space E is defined. This approach allows the investigation of cone spaces also in the case where the cone is not necessarily normal, which was not possible before 2007. Since then, there have appeared many papers containing interesting fixed point results in cone metric spaces. In this work, we extend Cantor's intersection theorem to cone metric spaces and derive, as an application, a fixed point theorem for mappings with locally power diminishing diameters, which were introduced by Istrăţescu [2].

Let E be a real Banach space and P a subset of E. P is called a cone if and only if:

(i) P is closed and non-empty, and $P \neq \{\theta\}$;
(ii) $a, b \in \mathbb{R}$, $a, b \geq 0$ and $x, y \in P$ imply $ax + by \in P$;
(iii) $P \cap (-P) = \{\theta\}$.

Given a cone $P \subset E$, we define a partial ordering $\preceq$ on E with respect to P by $x \preceq y$ if and only if $y - x \in P$. We shall write $x \prec y$ to indicate that $x \preceq y$ but $x \neq y$, while $x \ll y$ will stand for $y - x \in intP$ (the interior of P).

A cone $P \subset E$ is called normal if there is a number $K \geq 1$ such that for all $x, y \in E$,

$$\theta \preceq x \preceq y \quad \text{implies } \|x\| \leq K\|y\|.$$

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