JKAU: Eng. Sci., vol. 7, pp. 29-40 (1415 A.H. / 1995 A.D.)

Accurate Three-Term Approximation of the Angular Distribution of the Radiation Intensity

OMAR M. ALI AL-RABGHI
Thermal Engineering Department, Faculty of Engineering, King Abdulaziz University, Jeddah, Saudi Arabia

ABSTRACT. A very accurate approximation for the exponential integral $E_2(\kappa)$ was achieved by approximating the thermal radiation intensity by the first six terms of Legendre polynomial. The six constants in the three-term expression for $E_2(\kappa) = \sum_{i=1}^{3} a_i e^{-b_i \kappa}$ through algebraic manipulations were found. Unlike curve fitting approximations for $E_2(\kappa)$, the present three-term approximation gives accurate values for all optical thicknesses. For the sake of testing the new developed approximation, radiative equilibrium for planar participating gray media is considered. The radiative heat flux and the temperature distribution are compared with P-1 and P-3 method results. Using the developed approximation for attacking radiative heat transfer in nongray medium is illustrated.

1. Introduction

Thermal radiation heat transfer in participating media is of great importance for engineering as well as industrial applications. Due to the nature of radiation as act at a distance phenomenon, the radiation heat flux depends on the angular distribution of the intensity. Therefore, solid angle integration of the radiation intensity must be made. There are several methods available for solving radiative heat transfer problems in participating media. Howell[1] recently have presented a rather comprehensive review of these methods and drawback(s) of each. Among the method of choice are: The zone method[2,3], P-N method[4,5], two flux method[6,7], and others[1]. The solid angle integration usually shows in the solution of the transfer equation[8,9] in terms of the exponential integral $E_n(\kappa)$.

Accurate exponential approximation of $E_2(\kappa)$ and $E_3(\kappa)$ is needed for solving radiation heat transfer problems in nongray media, specially if the spectral absorption coefficient is changing continuously with wavelength. Soot and other semitransparent material exhibit such kind of behavior[10,11] therefore, gray medium assumption may be invalid in these cases.

Two distinct approaches are followed in approximating the exponential integral. The first is totally mathematical curve fitting[12,13], and the other is based on approximating the angular distribution of the radiation intensity[14]. The second approach gives reasonable accurate values for the two limiting cases of the optical thickness. The one-term and the two-term approximations using the second approach were made in Reference [14].

In this study, the procedure outlined by Reference [14] for one and two-term approximation is followed. The resulted three-term approximation is compared with other approximations. The goal of this work is to present a more accurate exponential approximation for $E_2(\kappa)$ and $E_3(\kappa)$ that can be used to predict radiative heat flux for gray and nongray participating media.

Another aspect of this study is to put the developed three-term approximation into work. Radiative equilibrium in gray planar media which has already been solved by different methods[5,15] is solved using the derived three-term approximation of the exponential integral, and comparison is made.

2. Mathematical Procedure for the Approximation

The procedure is based on expressing the angular distribution of the radiation intensity in terms of Legendre Polynomials truncated to m terms. Successive integration of the intensity moments will result in a differential equation expressing the jth differential of the heat flux. The differential of the heat flux to the same order can be found by differentiating the heat flux equation with no account for the boundaries and substituting the expression $E_2(\kappa) = \sum_{i=1}^{3} a_i e^{-b_i \kappa}$. Equating the coefficients of the two equations give the desire values of the constants a_i's and b_i's.

The spectral radiative transfer equation for 1-dimensional absorbing, emitting and non-scattering media is given by Reference [8].

$$\mu \frac{d i}{d \kappa} = i_b - i \tag{1}$$

where i is the radiation intensity at optical thickness κ, and as Fig. 1 shows $\mu = \cos\theta$. The radiative heat flux with no account for the boundaries can be found from Eq. (1) as

$$q_r(\kappa) = 2\pi \int_0^{\kappa} i_b(\kappa^*)\, E_2(\kappa - \kappa^*)\, d\kappa^* - 2\pi \int_0^{\kappa_L} i_b(\kappa^*)\, E_2(\kappa^* - \kappa)\, d\kappa^* \tag{2}$$

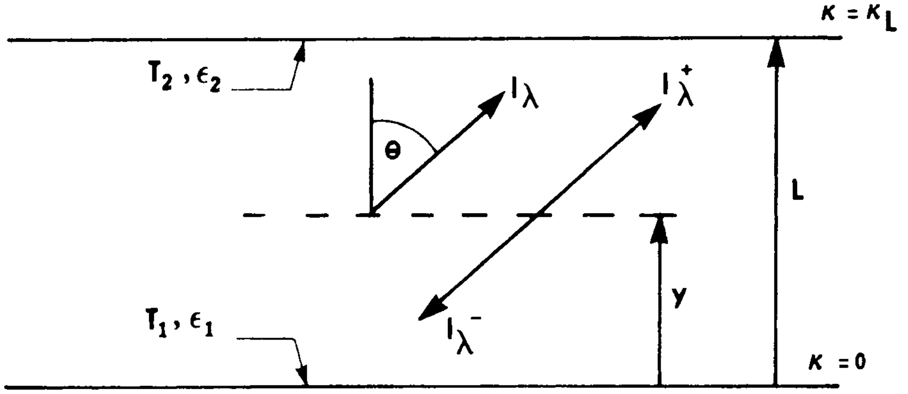

FIG. 1. Coordinate sytem for one-dimensional radiative transfer between 2 parallel plates.

where $E_n(\kappa)$ is called the exponential integral of order n[16], defined as

$$E_n(\kappa) = \int_0^1 \mu^{n-2} \exp(-\kappa/\mu)\, d\mu \tag{3}$$

Although exact values for $E_n(\kappa)$ are available in tabular form[16], but it is desire to have these functions (for easy spectral integration) written in exponential form, *i.e.* $\sum_i a_i e^{-b_i \kappa}$ specially if the spectral optical thickness κ continuously changes with wavelength.

The radiation intensity i is function of both κ and μ. To separate the two effects, the radiation intensity can be expressed in terms of Legendre Polynomial[16], *i.e.*

$$i(\kappa, \mu) = \frac{1}{4\pi} \sum_{n=0}^{\infty} (2n+1) A_n(\kappa) P(\mu) \tag{4}$$

where the integration of the intensity-polynomial product over the solid angle ω gives the coefficient A_n as follows

$$A_n(\kappa) = \int_{4\pi} i(\kappa, \mu) P(\mu)\, d\omega \tag{5}$$

$P(\mu)$ are the well known Legendre polynomial[16].

Equation (1) when multiplied by $(2n+1) P(\mu)$ and integrated over the solid angle we can arrive to the following expression relating A_{n-1}, A_n, A_{n+1} and i_b

$$(n+1)\frac{dA_n}{d\kappa} + n\frac{dA_{n-1}}{d\kappa} = -(2n+1)A_n + 4\pi\, i_b\, \delta_{On} \tag{6}$$

As indicated by Reference [14], only the first three A_n's values have physical meaning since they can be related to radiation energy, radiation heat flux, and radiation pressure respectively. Here we are only concerned with A_1, *i.e.*, the radiation heat flux.

$$A_1(\kappa) = \int_{4\pi} \mu\, i(\kappa, \mu)\, d\omega \tag{7}$$

Terminating the series given by Eq. (4) up to the A_5 , *i.e.*, $A_6 = 0$ and eliminating A_2 , A_3 , A_4 , and A_5 using equation (6). The resulted differential equation after quite lengthy algebra is put in the following form

$$\frac{d^6 q_r}{d\kappa^6} = \frac{1575}{75}\frac{d^4 q_r}{d\kappa^4} - \frac{4725}{75}\frac{d^2 q_r}{d\kappa^2} + \frac{315 * 11}{75} q_r$$

$$4\pi \frac{d^5 i_b}{d\kappa^5} - 4\pi \frac{882}{75}\frac{d^3 i_b}{d\kappa^3} + 4\pi \frac{105 * 11}{75}\frac{d i_b}{d\kappa} \tag{8}$$

The three-term approximation expansion for the exponential integral is wrıtten as

$$E_2(\kappa) = \sum_{i=1}^{3} a_i\, e^{-b_i\kappa} = a_1 e^{-b_1\kappa} + a_2 e^{-b_2\kappa} + a_3 e^{-b_3\kappa} \tag{9}$$

This expansion is substituted in the radiative flux equation, *i.e.*, Eq. (2) and six times differentiating for q_r is carried out. The resulted equation with some algebraic manipulation can be put in the same form as that of Eq. (8), *i.e.*

$$\frac{d^6 q_r}{d\kappa^6} = P\frac{d^4 q_r}{d\kappa^4} + Q\frac{d^2 q_r}{d\kappa^2} + S\, q_r + 4\pi\frac{d^5 i_b}{d\kappa^5}(a_1 + a_2 + a_3)$$

$$+ 4\pi\frac{d^3 i_b}{d\kappa^3}\left\{ (a_1b_1^2 + a_2b_2^2 + a_3b_3^2) - P(a_1 + a_2 + a_3) \right\}$$

$$+ 4\pi\frac{d i_b}{d\kappa}\left\{ (a_1b_1^4 + a_2b_2^4 + a_3b_3^4) - P(a_1b_1^2 + a_2b_2^2 + a_3b_3^2) \right.$$

$$\left. - Q(a_1 + a_2 + a_3) \right\} \tag{10}$$

where

$$\left.\begin{aligned} P &= b_1^2 + b_2^2 + b_3^2 \\ Q &= -(b_1^2b_2^2 + b_1^2b_3^2 + b_2^2b_3^2) \\ S &= b_1^2b_2^2b_3^2 \end{aligned}\right\} \tag{11}$$

Equating the coefficients of equation (8) and (10) and solving a cubic equation for b_1 , b_2 and b_3 , the following values for constants are found.

$$\left.\begin{aligned} b_1^2 &= 17.5625 \qquad & b_2^2 &= 1.1501 \qquad & b_3^2 &= 2.2873 \\ a_1 &= 0.4679 & a_2 &= 0.1714 & a_3 &= 0.3607 \end{aligned}\right\} \tag{12}$$

The three-term approximation for the exponential integral then becomes :

$$E_2(\kappa) = 0.4679\, e^{-4.1908\,\kappa} + 0.1714\, e^{-1.0724\,\kappa} + 0.3607\, e^{-1.5124\,\kappa} \tag{13}$$

The other order of the exponential integral, *i.e.*, $E_n(\kappa)$, $n \geq 2$ can be found using the relation[16]

$$\int E_n(\kappa)\, d\kappa = -E_{n+1}(\kappa) \tag{14}$$

For $n = 2$ we have

$$E_3(\kappa) = 0.1116\, e^{-4.1908\,\kappa} + 0.1598\, e^{-1.0724\,\kappa} + 0.2385\, e^{-1.5124\,\kappa} \tag{15}$$

3. Comparison between One, Two, and Three-Term Approximations

The one-term and two-term approximations of $E_2(\kappa)$ are derived and given by Reference [14] as

$$E_2(\kappa) = e^{-\sqrt{3}\,\kappa} \tag{16}$$

$$E_2(\kappa) = 0.348\, e^{-1.1613\,\kappa} + 0.652\, e^{-2.941\,\kappa} \tag{17}$$

Figure 2 shows qualitatively the difference between the three approximations in comparison with the exact values[16]. As the figure indicates, the difference between the approximations is more pronounced for small values of κ (*i.e.*, optically thin limit). For optically thick limit the exact value of $E_2(\kappa)$ is very small, and the three approximations tend to give accurate values for $E_2(\kappa)$.

For more comprehensive comparison between different exponential integral approximations, Table 1 is generated. Since in 1-D radiation heat transfer problems both $E_2(\kappa)$ and $E_3(\kappa)$ encountered, the table lists the percentage difference with the exact values for both $E_2(\kappa)$ and $E_3(\kappa)$. The table shows a fundamental conclusion about the accuracy of different techniques used to approximate the exponential integral. Unlike mathematical fitting, the three approximations given by Eqs. (16), (17) and (13) predict accurate values for both $E_2(\kappa)$ and $E_3(\kappa)$ for both limits of κ. The accuracy is increased as the order of the approximation is increased. It is also to be noted from Table 1 that although $E_2(\kappa)$ and $E_3(\kappa)$ exact values are small at large κ, but the percentage difference for curve fitting approximation is quite high.

4. Application to Gray Medium in Radiative Equilibrium

Gray participating media between two black parallel plates having temperatures T_1 and T_2 is considered, and the two exponential integral approximations given by Eq. (13) and Eq. (17) are individually tested. The geometry of the problems is shown in Fig. 1. For gray nonscattering media in radiative equilibrium, the energy equation for 1-D pure radiation and the radiative heat flux are given by

$$\nabla \cdot q_r = 0$$

$$= 2\,\sigma T_1^4\, E_2(\kappa) + 2\,\sigma T_2^4\, E_2(\kappa_L - \kappa)$$

$$+ 2\int_0^{\kappa_L} \sigma T^4(\kappa^*)\, E_1(\kappa - \kappa^*)\, d\kappa^* - 4\,\sigma T^4(\kappa) \tag{18}$$

$$q_r = 2\left[\sigma T_1^4 E_3(\kappa) - \sigma T_2^4 E_3(\kappa_L - \kappa) + \int_0^{\kappa} \sigma T^4(\kappa^*)\, E_2(\kappa - \kappa^*)\, d\kappa^*\right.$$

$$\left. - \int_0^{\kappa_L} \sigma T^4(\kappa^*)\, E_2(\kappa^* - \kappa)\, d\kappa^*\right] \tag{19}$$

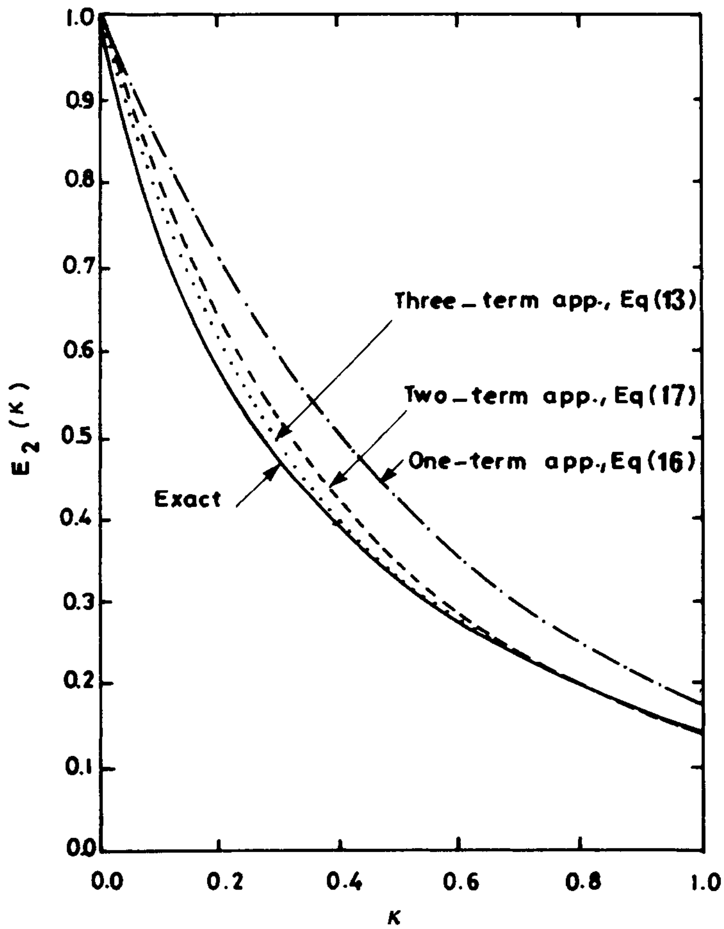

FIG. 2. Comparison of exact and exponential approximations of $E_2(\kappa)$.

TABLE 1. Comparison of different approximations for the exponential integral $E_2(\kappa)$ and $E_3(\kappa)$.

Ref.	[16]		[8]		[5]		[14]		[14]		Present	
	Exact		$a_1 = 0.90$ $b_1 = 1.8$		$a_1 = 0.75$ $b_1 = 1.5$		One-term Eq. (16)		Two-term Eq. (17)		Three-term Eq. (13)	
κ	$E_2(\kappa)$	$E_3(\kappa)$	PDE_2	PDE_3	PDE_2	PDE_3	PDE_2	PDE_3	PDE_2	PDE_3	PDE_2	PDE_3
0.0	1.0000	0.5000	10.0	0.0	25.0	0.0	0.0	- 15.5	0.0	- 4.3	0.0	- 2.0
0.02	0.9131	0.4810	4.9	- 0.3	20.3	- 0.9	- 5.8	- 16.0	- 4.6	- 4.3	- 3.8	- 2.0
0.04	0.8535	0.4633	1.9	- 0.4	17.2	- 1.3	- 9.3	- 16.3	- 6.8	- 4.3	- 5.4	- 1.9
0.06	0.8040	0.4468	- 0.5	- 0.5	14.8	- 2.3	- 12.1	- 16.5	- 8.3	- 4.2	- 6.2	- 1.7
0.08	0.7610	0.4311	- 2.4	- 0.4	12.6	- 2.9	- 14.4	- 16.6	- 9.4	- 4.0	- 6.6	- 1.6
0.1	0.7225	0.4163	- 4.0	- 0.3	10.7	- 3.4	- 16.4	- 16.6	- 10.1	- 3.8	- 6.8	- 1.4
0.2	0.5742	0.3519	- 9.4	0.9	3.2	- 5.2	- 23.2	- 16.0	- 11.1	- 2.5	- 5.8	0.4
0.4	0.3894	0.2573	- 12.5	5.4	- 5.7	- 5.2	- 28.5	- 12.2	- 7.8	0.2	- 1.7	0.8
0.6	0.2762	0.1916	- 10.7	11.4	- 10.4	- 6.1	- 28.1	- 6.6	- 3.2	2.2	1.0	1.2
0.8	0.2009	0.1443	- 6.2	17.9	- 12.5	- 4.3	- 24.6	- 0.1	0.7	3.4	2.1	1.1
1.0	0.1485	0.1097	- 0.2	24.7	- 12.7	- 1.7	- 19.1	- 6.9	3.4	3.8	2.2	0.7
1.2	0.1111	0.0839	6.6	31.3	- 11.6	1.5	- 12.6	13.9	5.1	3.6	1.8	0.3
1.4	0.0839	0.0646	13.7	37.7	- 9.5	5.2	- 5.5	20.9	5.7	3.1	1.1	- 0.1
1.6	0.0638	0.0499	20.8	43.8	- 6.6	9.2	1.9	27.6	5.7	2.3	0.5	0.4
1.8	0.0488	0.0387	27.8	49.4	- 3.3	13.2	9.3	34.0	5.1	1.4	0.0	- 0.5
2.0	0.0375	0.0301	34.5	54.7	0.5	17.4	16.6	40.0	4.3	0.5	- 0.4	- 0.6

The procedure to solve these two equations for the local temperature and heat flux is iterative in nature. The governing equations are solved two times. Once with the two-term exponential approximation given by Eq. (17), and another time with the newly developed three-term approximation, Eq. (13). As first trial the approximate temperature profile generated in Reference [17] is used for faster convergence. IBM-AT was used to solve the problem numerically. The results for the dimensionless radiative heat flux in comparison with reported P-1, P-3[5], and Reference [15] results are summarized in Table 2. The table shows the results for thin, intermediate and thick optical thicknesses. The dimensionless temperature for $\kappa_L = 1$ is listed in Table 3. The numerical results in the two tables illustrate the expected outcome that

TABLE 2. Comparison for the radiative heat transfer $\bar{q}_r$ for gray medium between two parallel black plates, $T_1 = 1500$ K, $\phi_2 = 0.5$.

	$\kappa_L = 0.1$	$\kappa_L = 1.0$	$\kappa_L = 10.0$
Exact^a	0.8585	0.5188	0.1095
P-1^b	0.8871	0.5357	0.1103
P-3^b	0.8641	0.5210	0.1095
2-term app.	0.8934	0.5250	0.1050
3-term app.	0.8732	0.5210	0.1087

a : Results are from Ref. [15].
b : P-1 and P-3 results from Ref. [5].

TABLE 3. Comparison for the dimensionless temperature $\phi = T/T_1$, $\kappa_L = 1.0$, $T_1 = 1500$ K, $\phi_2 = 0.5$ (Radiative equilibrium).

κ/κ_L	Ref. [15]	P-3	2-term app.	3-term app.
0.1	0.9377	0.9340	0.9321	0.9375
0.2	0.9031	0.9036	0.9051	0.9050
0.5	0.8537	0.8537	0.8537	0.8537
0.8	0.7939	0.7930	0.7910	0.7914
1.0	0.7334	0.7411	0.7447	0.7392

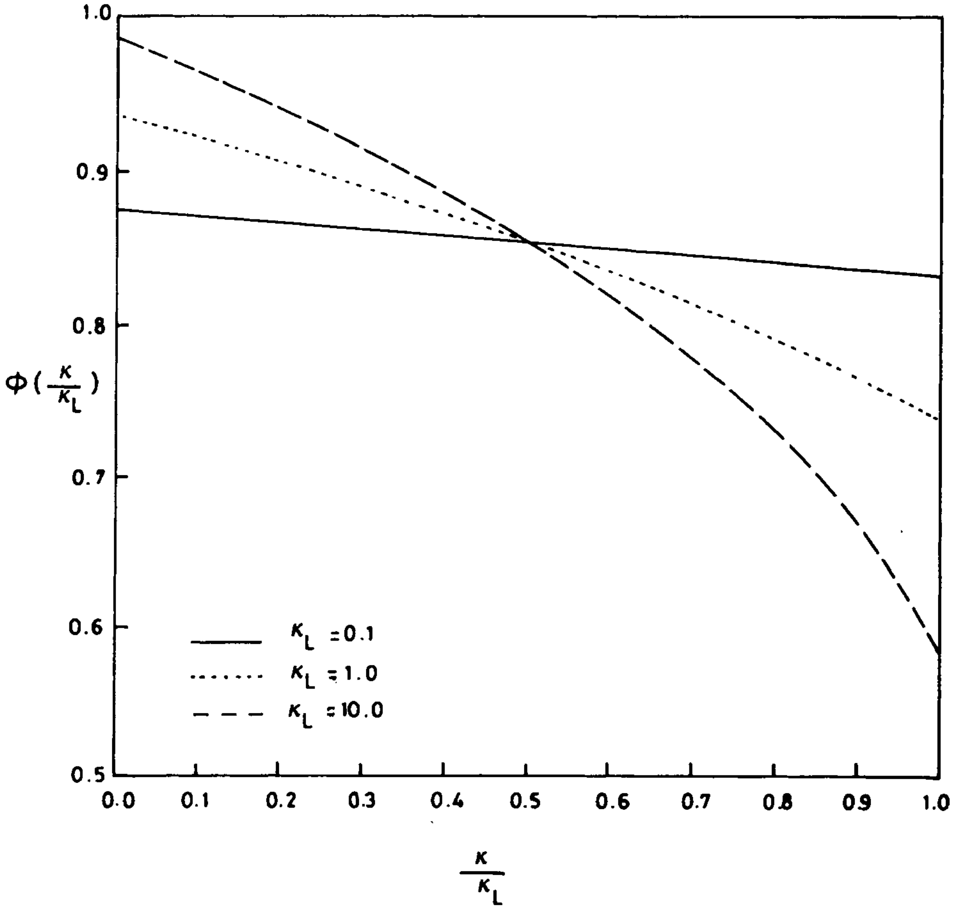

FIG. 3. Dimensionless temperature for gray planar media in radiative equilibrium.

the three-term approximation for the exponential integral predict results that are very close to the exact values, and therefore is recommended for use in cases were accurate solution for radiation heat transfer in 1-D gray or nongray participating media is desire. The dimensionless temperature profiles for $\kappa_L = 0.1$, $\kappa_L = 1.0$, $\kappa_L = 10$ generated using the three-term approximation are shown in Fig. 3. The difference between these curves and P-3[5] curves is unnoticeable. Table 2, Table 3 and Fig. 3 therefore, demonstrate that using the new approximation, accurate results for the radiative flux and temperature profiles are achieved.

In case the solution for radiative heat transfer for nongray medium is desire, the spectral absorption coefficient can be integrated easily over the wavelength using the developed three-term approximation for $E_2(\kappa)$ and $E_3(\kappa)$. If the spectral absorption coefficient can be assumed to change linearly with the wave number as

$$\kappa = \alpha_1 + \alpha_2 \frac{d}{\lambda} \tag{20}$$

where α_1, α_2 and d are constants. A typical spectral integral term of the following type will result

$$\int_{\lambda=\lambda_1}^{\lambda=\lambda_2} (\alpha_1 + \alpha_2 \frac{d}{\lambda})\, i_b \exp(-c(\alpha_1 + \alpha_2 \frac{d}{\lambda}))\, d\lambda \tag{21}$$

where λ_1 and λ_2 are wavelengths and c is constant.

This type of integration can be treated now more easily using the developed three-term approximation and special exponential integral function[16] without the worry that the result will not be accurate.

5. Conclusion

The exponential integral $E_2(\kappa)$ and $E_3(\kappa)$ encountered in the solution of engineering radiative heat transfer problems is approximated by three exponential terms. The approximation was found – in comparison with other approximations – to give very close values for $E_2(\kappa)$ and $E_3(\kappa)$.

Radiative equilibrium for gray medium between two black parallel plates are considered to put the developed approximation into work. The comparison with exact, P-1 and P-3 results illustrates that the three-term approximation predict the radiative heat transfer and the medium temperature accurately. Results are very close to P-3 prediction.

One of the real challenge in radiative problems is to solve for nongray medium. The developed approximation is very useful and accurate for in attacking such problems. The spectral integration over the wavelength or frequency can be made and incorporation of terms like $\exp(-\kappa_\lambda)$ can be easily carried out.

Nomenclatures

A_n	coefficient defined by Eq. (5).
a	absorption coefficient.
a_i	constant, as defined by Eq. (9).
b_i	constant, as defined by Eq. (9).

$E_n(\kappa)$	exponential integral, defined by Eq. (3).
$E_{2,ex}$	exact value for E_2
$E_{3,ex}$	exact value for E_3
I, i	thermal radiation intensity.
i_b	black body radiation intensity.
P	parameter, as defined by Eq. (11).
P_n	Legendre Polynomial.
PDE_2	percentage difference in $E_2(\kappa)$, $PDE_2 = \frac{E_{2,ex.} - E_{2,ap}}{E_{2,ex.}} * 100$
PDE_3	percentage difference in $E_3(\kappa)$, $PDE_3 = \frac{E_{3,ex.} - E_{3,ap}}{E_{3,ex.}} * 100$
q_r	radiative heat flux.
$\bar{q}_r$	dimensionless radiative heat flux, $q_r / (\sigma T_i^4)$
$\dot{Q}$	parameter, as defined in Eq. (11).
S	parameter, as defined in Eq. (11).
T_1 , T_2	temperature of the black plates, $T_1 = 1500$ K, $T_2 = 750$ K.
α_1 , α_2	arbitrary constants.
δ_{On}	kronecker delta, $\delta_{On} = 1$ only if $n = 0$, otherwise $= 0$.
ϕ	dimensionless temperature, T / T_1.
κ	optical thickness, $\kappa = \int_0^y a\, dy^*$.
κ_L	total optical thickness, $\kappa_L = \int_0^L a\, dy^*$.
μ	$\mu = \cos\theta$
ω	solid angle.

References

[1] **Howell, J.R.**, Thermal radiation in participating media: The past, the present, and some possible futures, *ASME J. Heat Trans.*, **110**: 1220-1229 (1988).

[2] **Byun, K.H.** and **Smith, T.F.**, Development of the zone method for linearly-anisotropic scattering media, *J. Quant. Spectrosc. and Rad. Trans.*, **40**(5): 591-604 (1988).

[3] **Hottel, H.C.** and **Sarofim, A.**, *Radiative Transfer*, McGraw-Hill, New York, p. 256 (1967).

[4] **Ratzell III, A.C.** and **Howell, J.R.**, Two-dimensional radiation in absorbing-emitting media using the P-N approximation, *ASME J. of Heat Trans.*, **105**: 333-340 (1983).

[5] **Yucel, A.** and **Bayazitoglu, Y.**, P-N approximation for radiative heat transfer in nongray medium, *AIAA Journal*, **21**(8): 1196-1203 (1983).

[6] **Wang, K.Y.** and **Tien, C.L.**, Radiative transfer through opacified fibers and powders, *J. Quant. Spectrosc. and Rad. Trans.*, **30**(3): 213-223 (1983).

[7] **Viskanta, R.**, Radiation heat transfer: Interaction with conduction and convection and approximate methods in radiation, *Proc. 7th Int., Heat Trans. Conf.*, **U. Grigull, E. Hahne, K. Stephan** and **J. Straub** (eds), Munich, **1**, pp. 103-122 (1982).

[8] **Siegel, R.** and **Howell, J.R.**, *Thermal Radiation Heat Transfer*, McGraw-Hill, p. 475, New York (1981).

[9] **Ozisk, M.N.**, *Radiative Transfer*, John Wiley, New York, p. 313 (1973).

[10] **Sarofim, A.F.** and **Hottel, H.C.**, Radiative transfer in combustion chambers: Influence of alternative fuels, *Sixth Int. Heat Transfer Conf., Toronto*, Aug. 1978, Hemisphere Publishing Corp., Vol. **6**, pp. 199-217 (1978).

[11] **Viskanta, R.** and **Anderson, E.E.**, Heat transfer in semitransparent solid, *Advances in Heat Transfer*, **11**: 317-441 (1975).

[12] **Cess, R.D.** and **Tiwari, S.N.**, Infrared Radiative Energy Transfer in Gases, *Advances in Heat Transfer*, **8**: 229-283, Academic Press, New York (1972).

[13] **Lick, W.**, Energy transfer by radiation and conduction, *Proceeding of the Heat Transfer and Fluid Mechanics Institute*, Stanford Univ. Press, Palo Alto, Calif., pp. 14-26 (1963).

[14] **Murty, S.S.**, Approximation on angular distribution of intensity of thermal radiation, *Int. Heat Mass Transfer*, **8**: 1203-1208 (1965).

[15] **Grosbie, A.L.** and **Viskanta, R.**, Interaction of heat transfer by conduction and radiation in a nongray planar medium, *Warme and Stoffubertragung*, **4**: 205-212 (1971).

[16] **Abramowitz, M.** and **Stegun, I.**, *Handbook of Mathematical Functions with Formulas, Graph and Mathematical Tables*, NBS Applies Mathematics Series, Dover Publications, New York, p. 227 (1965).

[17] **Dayan, A.** and **Tien, C.L.**, Heat transfer in gray planar medium with linear anisotrophic scattering, *J. Heat Transfer.*, **9**(3) August : 391-396 (1975).

علاقـــة تقريبيـــة دقيقـــة ذات ثلاثـــة حـــدود لإيجـــاد توزيـــع شـــدة الإشعـــاع الـــزاوي

عمر محمد علي شتيان الرابغي
قسم الهندسة الحرارية وتقنية تحلية المياه ، كلية الهندسة ، جامعة الملك عبد العزيز
جـــــدة - المملكة العربية السعودية

المستخلص . تم التوصل لمعادلة رياضية دقيقة لما يسمى بالتكامل المتناقص $E_2 = (\kappa)$ عن طريق تقريب شدة الإشعاع الحراري بالحدود الستة الأولى لمعادلة ليجندر ذات الحدود المتعددة . فلقد تم إيجاد الستة ثوابت في المعادلة $E_2 = (\kappa) = \sum_{i=1}^{3} a_i e^{-b_i \kappa}$ بحلول معادلات جبرية . وخلافًا لطريقة التثبيت التقريبي لمنحني $E_2 = (\kappa)$ ، فإن المعادلة التي تم التوصل لها للتكامل المتناقص تعطي نتائج جيدة لكل قيم السماكة البصرية . ومن أجل اختبار مدى صحة العلاقة التقريبية المقترحة التي تم التوصل لها ، فقد استخدمت هذه العلاقة في حل انتقال الحرارة بالإشعاع في وسط رمادي في اتجاه خطي واحد . تمت مقارنة نتائج انتقال الحرارة وتوزيع درجات الحرارة مع النتائج المقابلة لها باستخدام طريقة P-1 و P-3 . كما تم التعرض لإمكانية استخدام العلاقة لوسط غير رمادي .