Stability of Stepped Columns

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ABSTRACT. Stability of stepped columns is investigated in this paper. All factors contributing to their behavior are addressed. The effects of :elastic restraint at junction of two shafts; aspect and inertia ratios; as well as the variation in axial forces are considered. The paper is concluded by providing a closed-form solution for stability of such columns.

1. Introduction

Stepped columns are common in buildings and bridges as well as in machine parts. Their stability is heavily influenced by the constraints that may exist at the junction of the two shafts comprising the column. In-plane elastic restraints at the column step can be exerted by crane-girder utilities in some cases. Whereas, in the out-of-plane direction, such restraints can be induced by the bracing systems usually provided in the longitudinal direction of industrial buildings. The design of such columns is complicated by a number of factors, the least of which is the tediousness associated with the determination of relevant effective length factor. Any rational assessment should exhibit the effects of end restraints, constraints at the column step, aspect ratio of column segments, distribution of axial loads in column shafts and ratio of in-plane flexural rigidity of column shafts.

Today, the overall solution of governing differential equations is obtained by matching the deflections and slopes at the junction of column and then applying the boundary conditions. Anderson and Woodward[1] have addressed this problem from this perspective and presented fifteen characteristic equations for five end fixity types. Their work was extended later on by Agrawal and Stafie[2] to

accommodate two other end fixity types. However, their approach calls for three different equations for each fixity type and does not account for the elastic restraints that usually exist at the column junction. Also, Timoshenko and Ger[3] presented eigenvalues for some special cases of stepped columns. A semi-analytical procedure for evaluating the axial buckling of elastic columns with step-varying profiles, but with constant axial force is proposed by Arbabi and Li[4]. Nevertheless, the comprehensive reviews of the state-of -art on tapered columns provided by Ermopoulos[5]; and by Banerjee and Williams[6], showed the lack of design curves and experimental data in this regard.

This paper provides closed-form solutions for stability of stepped columns regardless of ends and junction restraints. The solution is obtained using the stiffness formulation which is habitual to the thinking of all structural engineers. In this context, it is worth mentioning that El-Tayem[7] presented rigorous solutions for stability of stepped columns for five end fixity types. It is verified within the course of this paper that part of widely used numerical values for the K-factor as provided by Agrawal and Stafiej[2] have serious errors. Furthermore, the paper demonstrates the sensitivity of the K-factor to the elastic restraints at the junction of the two shafts.

2. Mathematical Modeling

Efficiency in design of stepped columns can be achieved whenever the two shafts making the column buckle simultaneously and exert no restraint on each other. This will occur if each shaft sustains an axial load that is proportional to its buckling load. Unfortunately, values of moment of inertia may be fixed by some considerations other than satisfying the aforementioned condition. Thus, one segment of the column will buckle, and it can be interpreted that the un-buckled segment provides elastic restraints on the buckled piece. El-Tayem and Goel[8]; and Goel and El-Tayem[9] have demonstrated, theoretically and experimentally, the dominant role of elastic restraints on buckling load capacity of compression elements during both elastic and inelastic excursions.

To formulate the problem , a stepped column of the shown configuration and ends as well as step restraints, (see Fig. 1) is considered. In pursuing the analytical solution of the adopted model, the following assumptions underlay the analysis process :

1. Torsional buckling pattern is precluded. Thus , the column buckles in pure flexural mode around one of its principal axes.
2. The system buckles in the elastic range.
3. Restraints exerted at the column step are idealized by the shown attached linear spring.
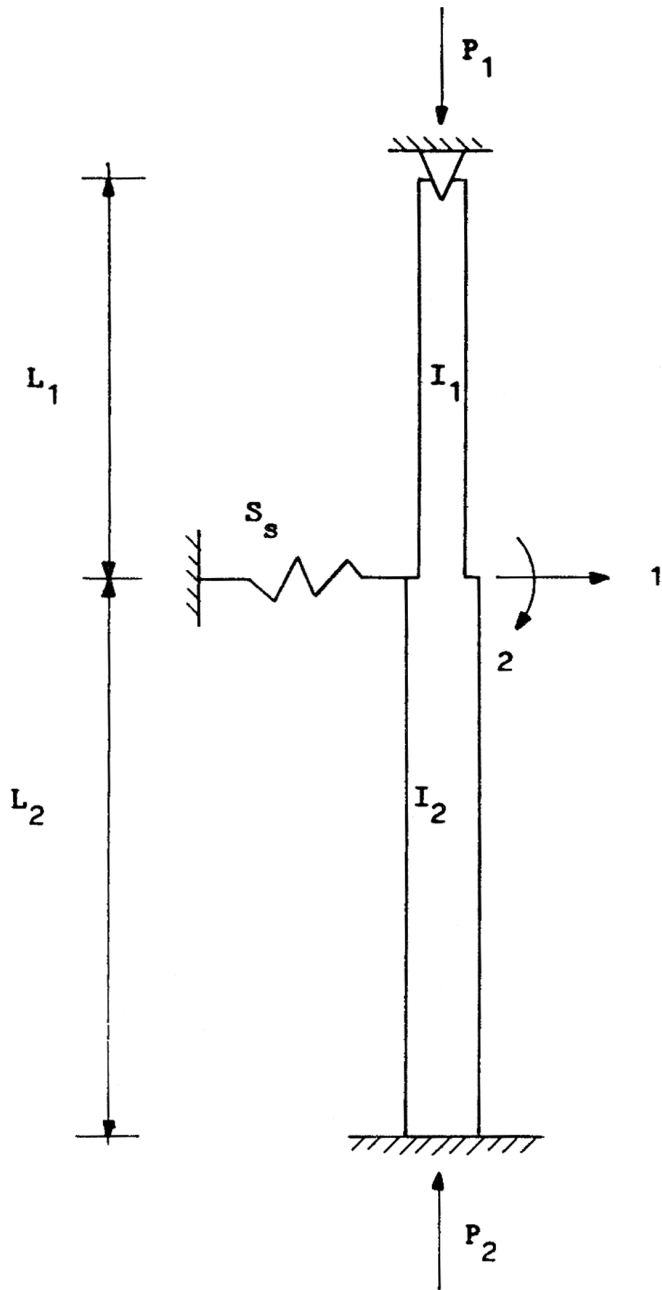4. Shear deformation and its effect on the stiffness of the system is ignored.

FIG. 1. Geometric layout and kinematics of stepped column.

5. Axial deformations are ignored.

6.   The axial force in each shaft is constant, which implies that the self-weight of column is ignored.

7.   The P-δ effect is considered by introducing the geometric stiffness term, S_g.

The configurations of the majority of cross-sections used in the fabrication of stepped columns endorse the validity of the first assumption as columns used have at least one axis of symmetry thus forcing the prevail of the flexural buckling mode. While the second assumption is quite frequently adopted in pursuing the stability problems, the move to inelastic buckling requires replacement of the modulus of elasticity, E, by the tangential moduls, E_t. The inclusion of lateral restraint at column step, assumption 3, is arbitrary and its value is subject to engineering judgment. If such restraint does not exist, the stiffness of the attached spring will be set equal to zero. Shear and axial deformations as well as the self-weight of the column in real structures rarely have any pronounced effect on the stability of the system nor on its behavior as such justifying assumptions 4, 5 and 6. It should be pointed out that these assumptions (4, 5 and 6) are normally part of any adopted realistic idealization model. Thus, one can conclude that the assumptions made are realistic and representative of actual behavior.

The kinematics at the junction of the two shafts in terms of the coordinates system as seen in Fig. 1 are defined by translational and rotational degrees of freedom. This idealization is valid regardless of end fixity type, because static condensation technique can be used to eliminate the non-essential degrees of freedom. The stiffness matrix of the system corresponding to the coordinates in Fig. 1 can be written as follows:

$$[S] = \begin{bmatrix} S_{11} + S_s - S_g & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \tag{1}$$

where the stiffness coefficients, S_{11}, S_{12}, S_{21} and S_{22}, hold the traditional definition, i.e. S_{ij} = nodal force at and in direction of coordinate i due to a nodal displacement at and in direction of coordinate j. The terms S_g and S_s denote, respectively, the offsetting transverse force in the direction of coordinate 1 due to p-δ effect and the stiffness of the linear spring lumped at the column step. It should be observed that as far as design is concerned, this modeling is valid whether the two shafts are symmetrically or asymmetrically positioned. This can be justified by referring to the interaction formulae of both the American Institute of Steel Construction Manual[10] and the American Concrete Institute Code[11] where it is clearly stated that buckling load capacity should be estimat-

ed for concentric columns. This means that the moment induced at the column step due to off-setting of the two shafts should be superimposed on to the external moment. Thus, it would affect only the flexural stress ratio.

The stability equation of the system at the inception of buckling can be written by:

$$[S] \ \{D\} = 0 \tag{2}$$

where $\{D\}$ represents the displacement vector associated with the adopted kinematics. The conditions under which Eq. 2 yields a displacement vector with at least one non-zero element constitute the static buckling conditions. A nontrivial solution is feasible only when the value of the determinant of the stiffness matrix vanishes. Expansion of S-matrix; yields the following general characteristic equation of the system:

$$S_{11}S_{22} + S_sS_{22} - S_gS_{22} - S_{12}S_{21} = 0 \tag{3}$$

Eq. 3 represents the buckling condition of stepped columns inspite of end and step restraints. Eigenvalues can be obtained using Eq. 3 once the stiffness coefficients are determined.

3. Local Stiffness Coefficients

The classical end rotational stiffness of the beam element as displayed in Fig. 2 needs modification to account for the presence of the compressive force, P. The end moment, M_2, required to rotate the near end A through unity while the far end B remains undisturbed is given by Maugh[12] as follows:

$$M = \frac{EI\emptyset}{L} \ [\ \frac{1 - \emptyset \cot \emptyset}{2 \ \tan \ \emptyset/2 - \emptyset} \ ] \tag{4}$$
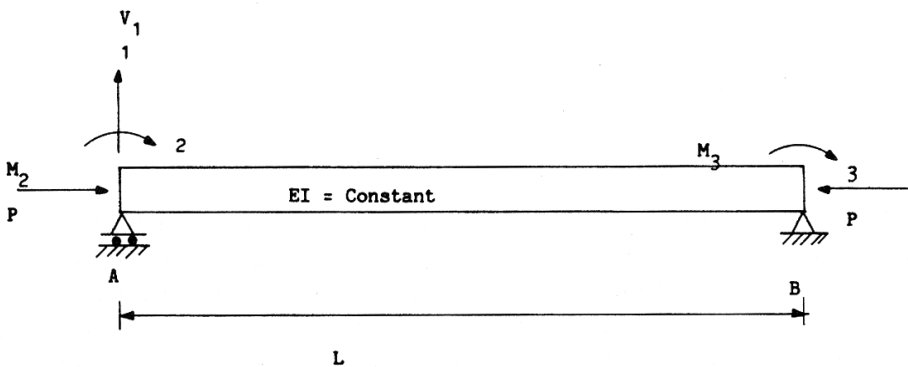

FIG. 2. Element nodal forces and displacements.

Instantaneously, a holding moment, M_3, will be developed at end B to maintain its compatibility of magnitudes given by

$$M_3 = \frac{EI\varnothing}{L}[\frac{-1 + \varnothing\csc\varnothing}{2\tan\varnothing/2 - \varnothing}] \tag{5}$$

where E, I, and L, respectively denote Young's modulus, moment of inertia and the unsupported length of the element. Whereas, the non-dimensional load parameter, $\varnothing$, is set equal to $\sqrt{PL^2/EI}$.

The structural system, as mentioned earlier, is idealized by two degrees of freedom. Thus, the moment given in Eq. 4 needs adjustment to account for other type of restraints that may exist at the far end of the element. Two types of supports, namely: guided (slider) and hinged supports, frequently exist either at the top or bottom of stepped columns. The adjusted expressions of end moment required to impose a unit rotation are obtained utilizing the condensation technique and are as follow:

$$M_2 = \frac{EI}{L}\varnothing \begin{cases} \dfrac{\varnothing}{1 - \varnothing\cot\varnothing} & \text{if, far end is hinged} \\[2em] \dfrac{1 - .5\varnothing\cot\varnothing/2}{2\tan\varnothing/2 - \varnothing} & \text{if, far end is slider} \end{cases} \tag{6}$$

The nodal force, V_1 required to force end A acquiring a nodal unit translation while suppressing all other displacement must be determined. Equilibrium of forces on element of Fig. 2 together with condensation technique are utilized to develop expressions for V_1 for different boundary conditions. These expressions are:

$$V_1 = \frac{EI}{L^3}\varnothing^2 \begin{cases} \dfrac{2\tan\varnothing/2}{2\tan\varnothing/2 - \varnothing} & \text{if, far end is fixed} \\[2em] \dfrac{1}{1 - \varnothing\cot\varnothing} & \text{if, far end is hinged} \\[2em] 0 & \text{if, far end is slider or free} \end{cases} \tag{7}$$

The geometric stiffness term, S_g, represents the tendency toward buckling. For this particular problem, it depends only on the configuration of the column and the distribution of axial forces in column shafts. Linear approximation, first-order analysis, is used to simulate the P-δ effect. Accordingly, the induced secondary moment $P \times 1$, must be balanced by an end couple, P/L, to maintain the element in equilibrium. Thus, the following expression is presented for S_g:

$$S_g \begin{bmatrix} \dfrac{P}{L} \\ 0 \quad \text{if, far end is slider} \end{bmatrix} \tag{8}$$

4. Characteristic Equations

Characteristic equations for broad range of end restraints that are often encountered in engineering practice, see Fig. 3, are developed. The synthesis process of these equations is carried out by substituting the stiffness coefficients in Eq. 3. The latter coefficients are assembled by adding the stiffness contribution of each shaft as given by Eqs. 4-8 to the global stiffness matrix, S.
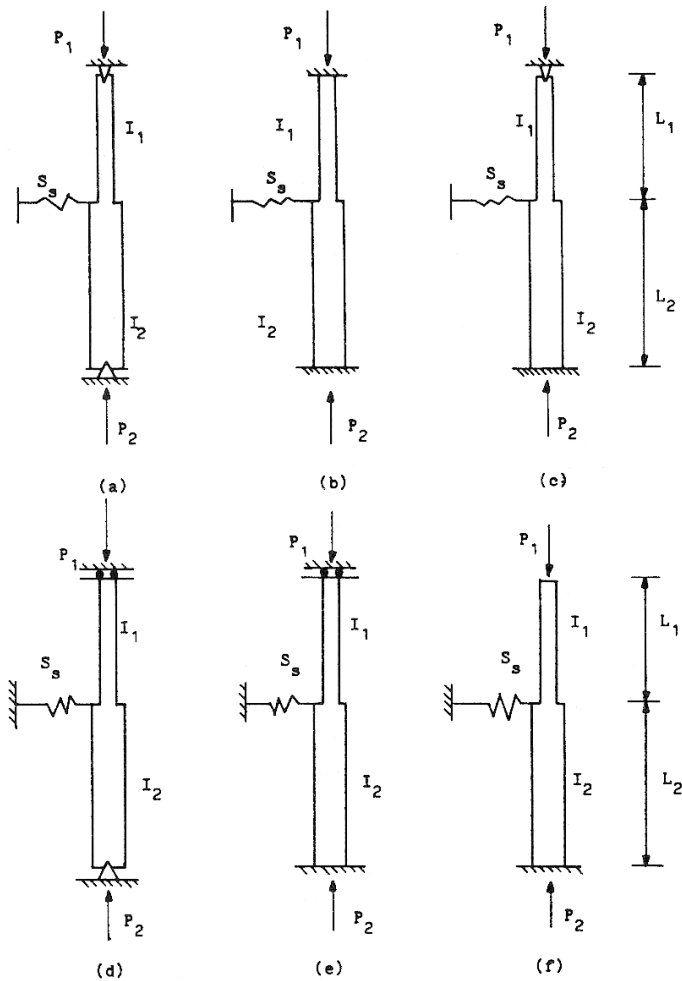

FIG. 3. Stepped column of different end restraints: (a) Pin-Pin, (b) Fix-Fix, (c) Fix-Pin, (d) Pin-Slider, (e) Fix-Slider and (f) Fix-Free.

For compact notation, the non-dimensional load parameters defined by

$\emptyset_1 = \sqrt{P_1 L_1^2 / E I_1}$ and $\emptyset_2 = \sqrt{P_2 L_2^2 / E I_2}$ are introduced, where:

$$\begin{aligned}
I_1 &= \text{moment of inertia of upper shaft;} \\
I_2 &= \text{moment of inertia of lower shaft;} \\
L_1 &= \text{length of upper shaft;} \\
L_2 &= \text{length of lower shaft;} \\
P_1 &= \text{axial load in upper shaft; and} \\
P_2 &= \text{axial load in lower shaft.}
\end{aligned}$$

Following the outlined procedure, simplification and arrangement of parameters, yield the following characteristic equations:

1 – Pinned-Pinned Case, Fig. 3-a

a – For $0 < P_1/P_2 \leq 1$

$$[1+\beta]^2 - \beta[\gamma + \beta(1-K_s)][1-\emptyset_2 \cot\emptyset_2] - [1+\frac{\beta}{\gamma}(1-K_s)][1-\emptyset_1 \cot\emptyset_1] = 0 \qquad (9)$$

b – For $P_1/P_2 = 0$

$$3[1+\beta]^2 + \frac{\beta^3}{\alpha}\emptyset_2^2[K_s - 1] - 3\beta^2[1-K_s][1-\emptyset_2 \cot\emptyset_2] = 0 \qquad (10)$$

2 – Fixed-Fixed Case, Fig. 3-b

a – For $0 < P_1/P_2 \leq 1$

$$\emptyset_2[2\tan\frac{\emptyset_2}{2}\emptyset_2][\emptyset_1 \cot\emptyset_1(\gamma - K_s\beta) + K_s\beta] + \frac{\emptyset_1^2}{\beta}[1-\emptyset_2 \cot\emptyset_2] +$$

$$\frac{\emptyset_1}{\gamma}[2\tan\frac{\emptyset_1}{2}\emptyset_1][\emptyset_2 \cot\emptyset_2(1-K_s) + K_s] + \emptyset_2^2\beta[1-\emptyset_1 \cot\emptyset_1] + \qquad (11)$$

$$2\emptyset_1\emptyset_2 \ \tan\frac{\emptyset_1}{2} \ \tan\frac{\emptyset_2}{2} = 0$$

b – for $P_1/P_2 = 0$

$$[\frac{24\alpha}{\emptyset_2^2\beta^2} + 12 + 8K_s\beta]\tan\frac{\emptyset_2}{2} + \emptyset_2\beta[4 + K_s\alpha - 4K_s]$$

$$+[\beta\emptyset_2^2(\frac{\beta}{\alpha} - K_s\alpha) - \frac{12}{\beta}]\cot\emptyset_2 + \frac{12}{\beta\emptyset_2}[1-\frac{\alpha}{\beta}] = 0 \qquad (12)$$

3 – *Fixed-Pinned Case, Fig. 3-c*

a – For $0 < P_1/P_2 \leq 1$

$$\varnothing_2[\gamma + \beta - \beta K_s + \frac{K_s}{\gamma \varnothing_2^2}] + 2[1 - \gamma + \beta K_s]\tan\frac{\varnothing_2}{2} + \frac{1 - K_s}{\gamma}\cot\varnothing_2$$

$$+ \frac{\varnothing_1}{\varnothing_2}[\frac{1}{\beta} - \frac{K_s}{\gamma}]\cot\varnothing_1 - \varnothing_1[\frac{1}{\gamma} + \frac{1}{\beta} - \frac{K_s}{\gamma}]\cot\varnothing_1\cot\varnothing_2 = 0 \tag{13}$$

b – For $P_1/P_2 = 0$

$$\frac{3}{\beta\varnothing_2} + \beta\varnothing_2[3 - 3K_s + \frac{\beta}{\alpha}K_s] + 6[1 + \beta K_s]\tan\frac{\varnothing_2}{2}$$

$$+[\frac{\beta^2}{\alpha}\varnothing_2^2(1 - K_s) - \frac{3}{\beta}]\cot\varnothing_2 = 0 \tag{14}$$

4 – *Pinned-Slider Case, Fig. 3-d*

a – For $0 < P_1/P_2 \leq 1$

$$\varnothing_1[1 - \varnothing_1\cot\varnothing_1 - \tan^2\frac{\varnothing_1}{2}][\cot\varnothing_2 + \frac{K_s}{\varnothing_2} - K_s\cot\varnothing_1]$$

$$+ \frac{\beta_{\varnothing_2}}{\alpha}[K_s - 1][2\tan\frac{\varnothing_1}{2} - \varnothing_1] = 0 \tag{15}$$

b – For $P_1/P_2 = 0$

$$K_s\beta[\frac{\beta}{\alpha}\varnothing_2 + \frac{1}{\varnothing_2}] + [1 - \beta K_s]\cot\varnothing_2 - \frac{\beta_{\varnothing_2}}{\alpha} = 0 \tag{16}$$

5 – *Fixed-Slider Case, Fig. 3-e*

a – For $0 < P_1/P_2 \leq 1$

$$\varnothing_1[2\tan\frac{\varnothing_{2-\varnothing_2}}{2}][1 - \varnothing_1\cot\varnothing_1 - \tan^2\frac{\varnothing_1}{2}][1 + \frac{2}{\varnothing_2}K_s\tan\frac{\varnothing_1}{2} - K_s]$$

$$+ \frac{\beta}{\alpha}[1 - K_s]\varnothing_2[2\tan\frac{\varnothing_{1-\varnothing_1}}{2}][1 - \varnothing_2\cot\varnothing_2 - \tan^2\frac{\varnothing_2}{2}] \tag{17}$$

$$+ \frac{\beta}{\alpha}K_s[2\tan\frac{\varnothing_{1-\varnothing_1}}{2}][2\tan\frac{\varnothing_{2-\varnothing_2}}{2}] = 0$$

b – For $P_1/P_2 = 0$

$$[2\tan\frac{\varnothing_{2-\varnothing_2}}{2}][1+\frac{\beta}{\alpha}K_s - K_s + \frac{2}{\varnothing_2}K_s \tan\frac{\varnothing_2}{2}]$$

$$+\frac{\beta}{\alpha}\varnothing_2[1-\varnothing_2 \cot\varnothing_2 - \tan^2\frac{\varnothing_2}{2}][1-K_s]=0 \tag{18}$$

6 – Fixed-Free Case, Fig. 3-f

a – For $0 < P_1/P_2 \le 1$

$$[2\tan\frac{\varnothing_{2-\varnothing_2}}{2}][\frac{K_s}{\gamma\varnothing_2}(1-\varnothing_2 \cot\varnothing_2)-\frac{\beta}{\varnothing_1}\tan\varnothing_1(2K_s \tan\frac{\varnothing_2}{2}-K_s\varnothing_2 + \varnothing_2)]$$

$$+\frac{1}{\gamma}[1-\varnothing_2 \cot\varnothing_2 - \tan^2\frac{\varnothing_2}{2}]=0 \tag{19}$$

b – For $P_1/P_2 = 0$

$$[1-\varnothing_2 \cot\varnothing_2][1+\frac{K_s}{\varnothing_2}(2\tan\frac{\varnothing_{2-\varnothing_2}}{2})]-\tan^2\frac{\varnothing_2}{2}=0 \tag{20}$$

The following definitions are assigned to the non-dimensional parameters:

$$K_s = S_s \frac{L_2}{P_2};$$

α = ratio of moments of inertia, I_1/I_2; of upper to lower shaft;

β = length ratio of upper to lower column segments, L_1/L_2; and

γ = ratio of axial forces, P_1/P_2 of upper to lower shaft

Also, the parameters $\varnothing_1$ and $\varnothing_2$ are interrelated as follows:

$$\varnothing_1 = \varnothing_2\beta\sqrt{\frac{\gamma}{\alpha}} \tag{21}$$

Eqs. 9-20 can be solved for the parameter $\varnothing_2$. Since only the first buckling load has any real significance, the equations are solved for their lowest roots. Thereafter, the effective length factor associated with elastic buckling of lower segment is attainable as follows:

$$K_2 = \frac{\pi}{\varnothing_2} \tag{22}$$

Meanwhile, Eq. 21 permits the direct determination of effective length factor of upper segment by:

$$K_1 = \frac{K_2}{\beta} \sqrt{\frac{\alpha}{\gamma}} \tag{23}$$

A computer program has been written to solve the characteristic equations for prescribed values of inertia, load, length ratios and spring stiffness.

5. Intermediate Spring

The variations of the effective length factor associated with the increase in the stiffness of intermediate spring attached at column step are shown in Figs. 4-9. The figures are plotted for the six boundary conditions given in Fig. 3. The inertia ratio, α, as well as the length ratio, β, are held constant with the view of triggering out, mainly, the influence of the spring rigidity on the stability of the system.
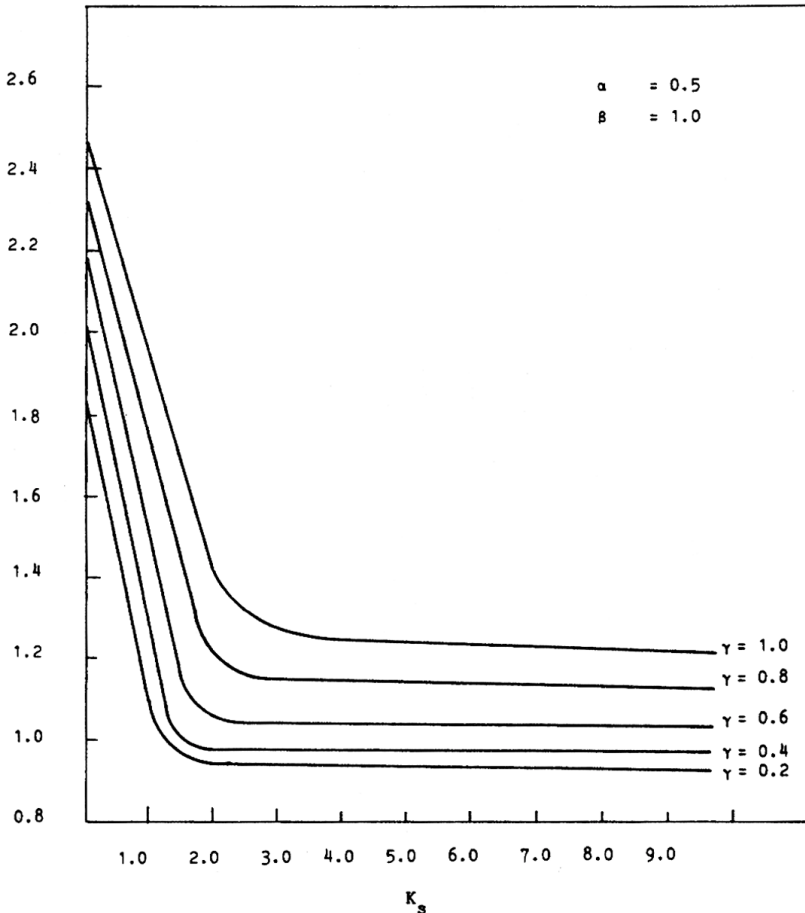

FIG. 4. The variation of K-factor with spring rigidity for Pin-Pin stepped column.
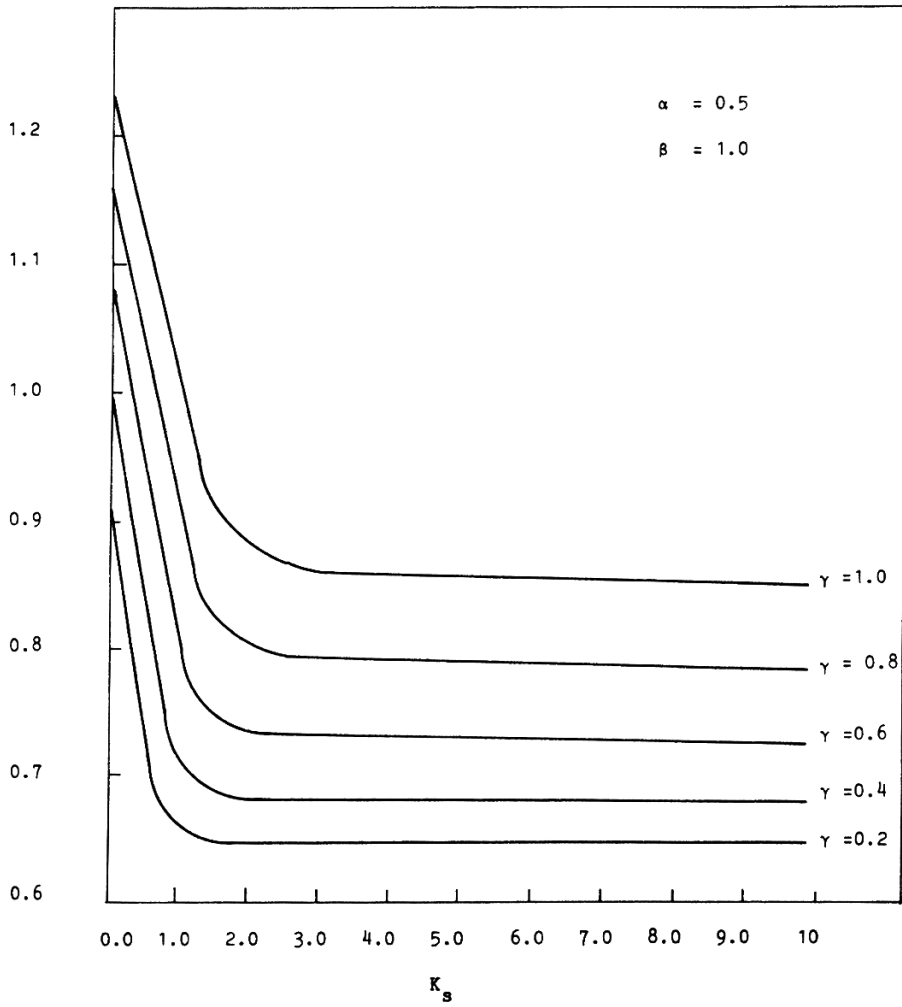
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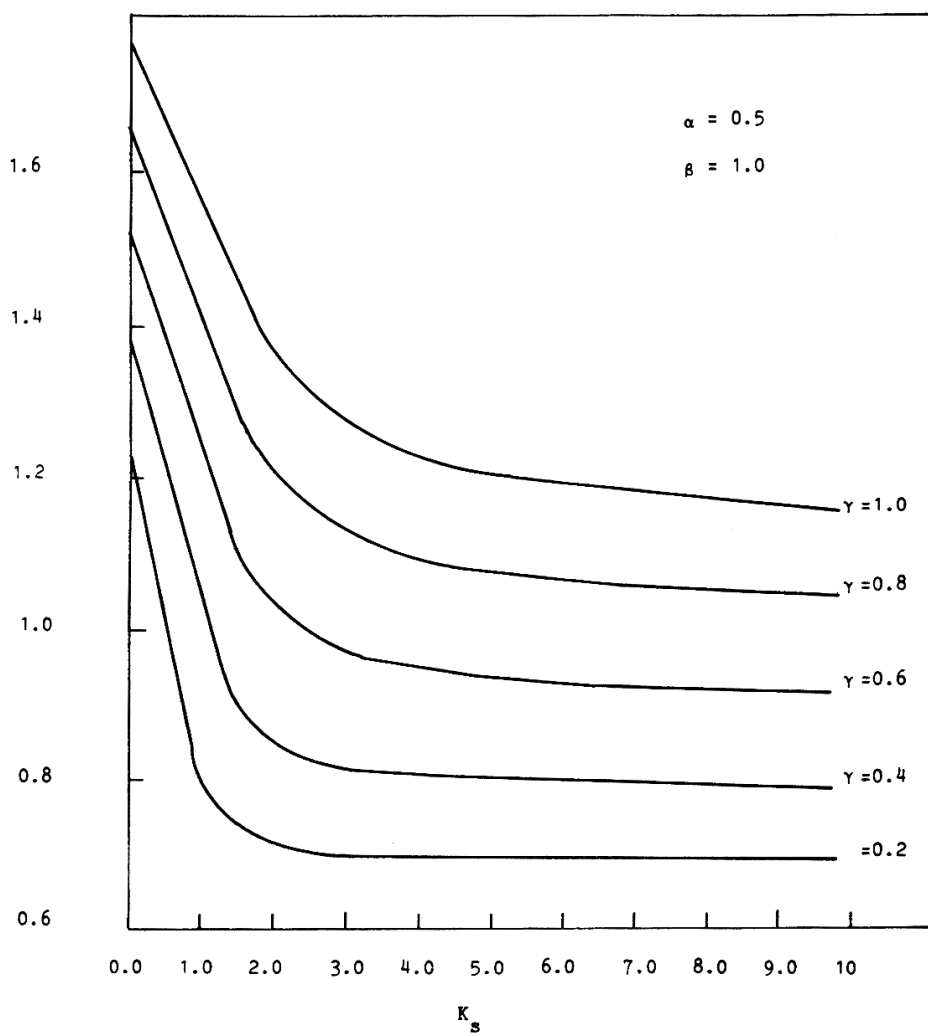FIG. 5. The variation of K-factor with spring rigidity for Fix-Fix Stepped Column.

Fɪɢ. 6. The variation of K-factor with spring rigidity for Fix-Pin Stepped Down.
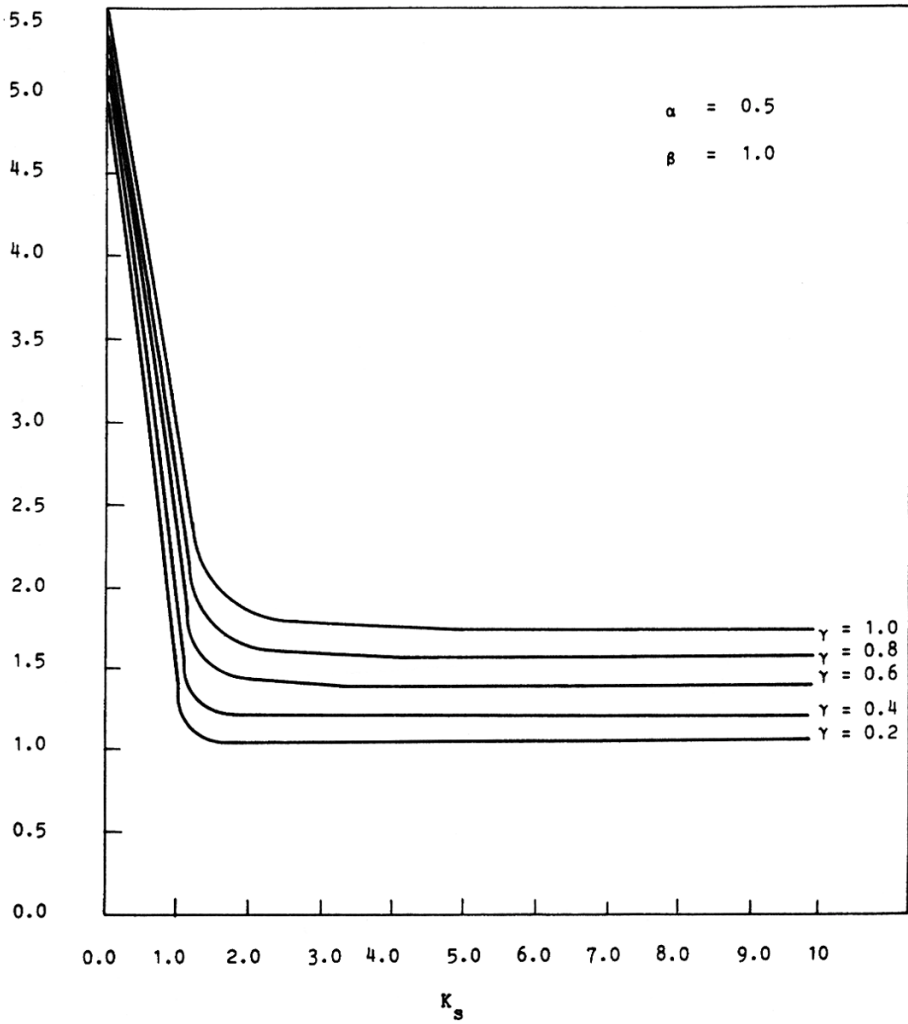
Fig. 7. The variation of K-factor with spring rigidity for Pin-Slider Stepped column.
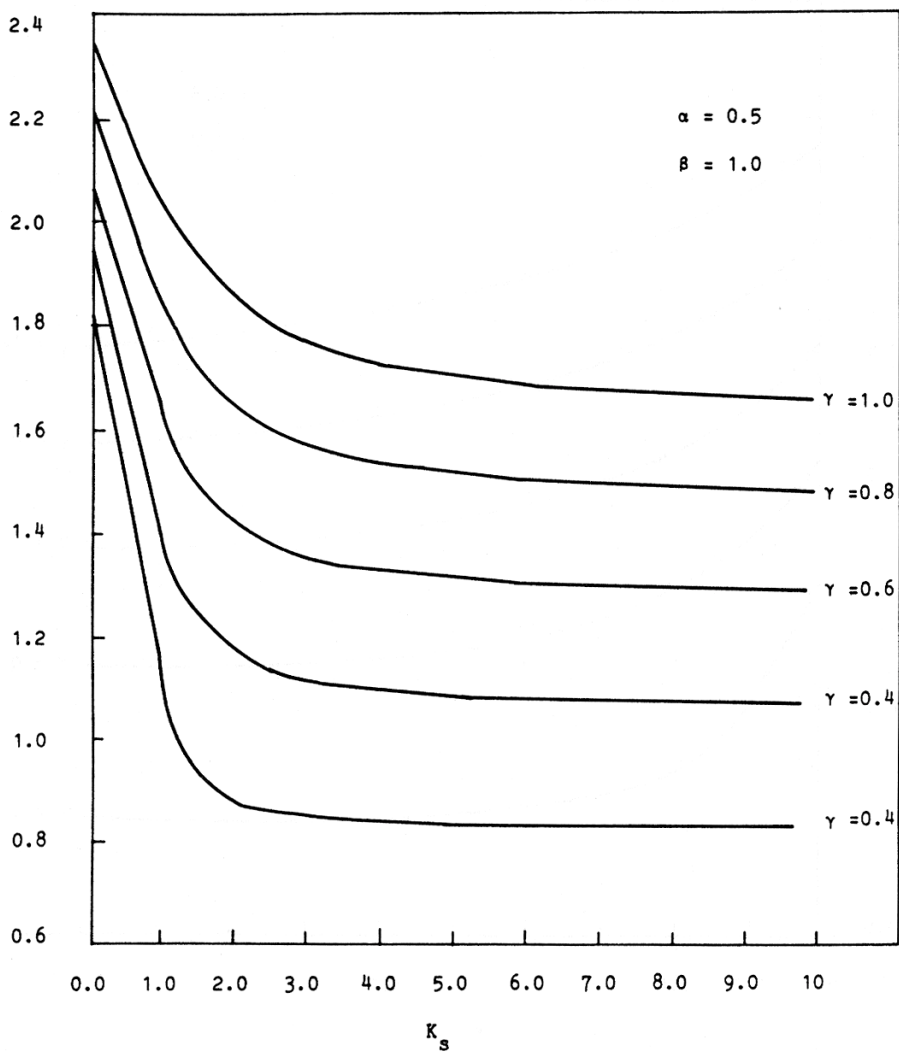
F<small>IG</small>. 8. The variation of K-factor with spring rigidity for Fix-Slider Stepped column.
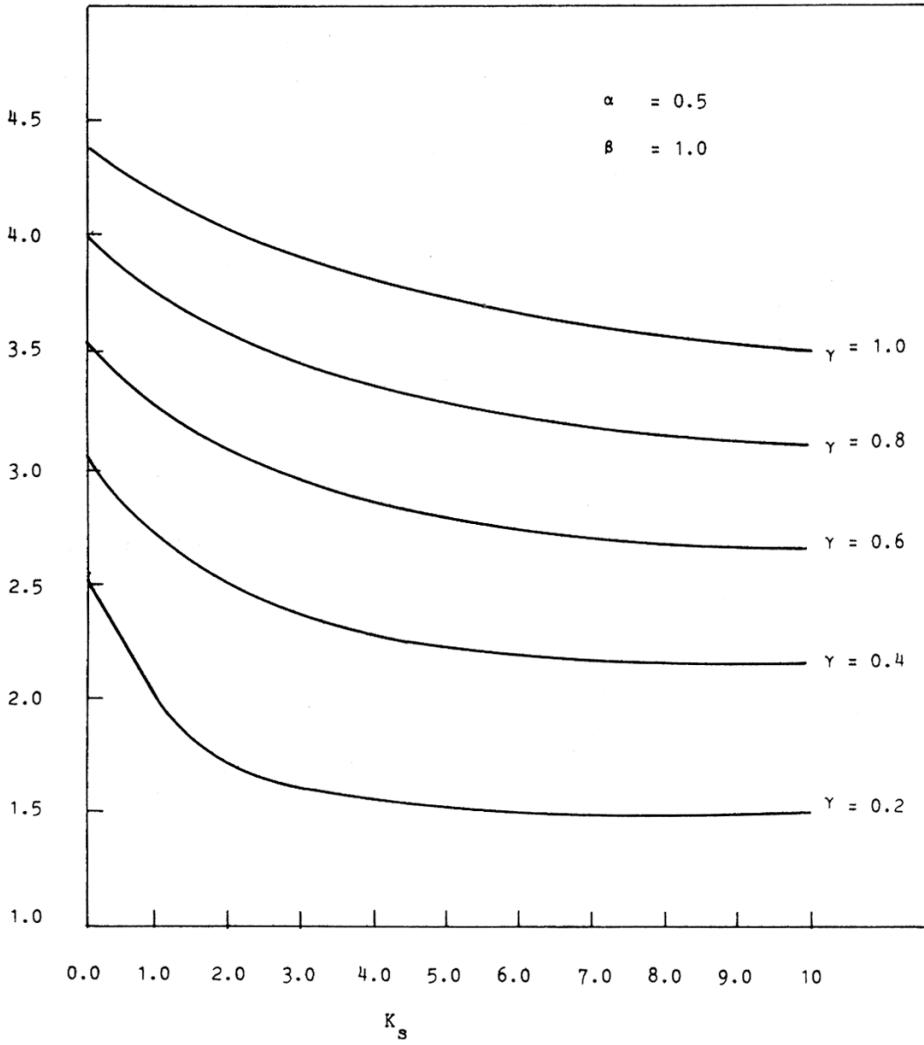
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F𝐼G. 9. The variation of K-factor with spring rigidity for Fix-Free Stepped column.

The results as displayed in Figs. 4-9 show the sweeping role of the intermediate spring in cutting-down the effective length factor, as such, enhancing the ability of the system as load carrier. Also, it can be easily detected from the figures that the effective length factor attains its stability at relatively low values of spring rigidity. Practically speaking, the K-factor maintains fixed value for all values of non-dimensional stiffness parameter, K_s, greater than 2. Exceptional is the Fix-Free case where stability of K-factor is attained at values of K_s greater than 5. This means that full lateral support at the column step is provided whenever the spring rigidity is in excess of $2P_2/L_2$. It is worth noting that the $2P_2/L_2$ can be satisfied only if minimum bracing is provided in the plane under consideration. Also, it is clearly seen that for the same loading and geometric conditions, the effect of the spring rigidity is more pronounced for the Pin-Slider, Fig 7, and the Fix-Free, Fig. 9, end restraints.

6. Verification

For testing the accuracy of the characteristic equations presented, some cases whose closed-form or semi-analytical solutions are available are discussed and the results are as follows :

Pinned-Pinned Members

For the Pin-Pin members of Fig. 3-a with rigid step spring and when the ratios, α, β, and γ, respectively, equal to 1, 0.5 and 1, Eq. 9 reduces to: $\varnothing_2(\cot\varnothing_2 + \cot\varnothing_2/2) - 3 = 0$. This equation has a solution at $\varnothing_2 = 3.8566$ and referring to Eqs. 20 and 21, K_1 and K_2 become 1.6292 and 0.8146, respectively. This result agrees with the result given by Timoshenko and Gere[3].

Whenever, the stiffness of the attached spring approaches zero and for the case where the non-dimensional parameters are set at: $\alpha = 0.5$; $\beta = 1$; and $\gamma = 1$, Eq. 9 yields to $\cot\varnothing_2 + 2\cot 2\varnothing_2 = 0$, which has its first eigenvalue at $\varnothing_2 = 0.95533$.

Therefore, K_1 and K_2 have values of 1.6443 and 3.2885, respectively. The latter values conform to within $\pm 0.1\%$ with those presented by Arbabi and Li[4].

With a zero-stiffness spring and values of 1, 1, and 0 for α, β, and γ, Eq. 10 shrinks to: $9 - \varnothing_2^2 + 3\varnothing_2 \cot\varnothing_2 = 0$. The first positive root for the equation exists at $\varnothing_2 = 2.1599$, hence, K_1 and K_2 become, respectively equal to ∞ and 1.4545. These results are in total agreement with those given by Shrivastava[13]; and Williams and Aston[14]. However, for the given loading and geometric conditions, Agrawal and Stafiej[2] assigned values of 0.0 and 1.45 for K_1 and K_2. This implies that their results are in error and need adjustment.

Fixed-Fixed Members

When the stiffness of the attached spring, Fig. 3-b approaches zero, while the parameters α, β, and γ are all set equal unity, Eq. 11 reduces to $1 - \varnothing_2 \cot \varnothing_2 = 0$, which has a solution at $\varnothing_2 = 4.492$, thus, K_1 and K_2 have the value of 0.6999. This result agrees with that given by and[3].

If values of 0.5, 0.5, 0 and 0 are, respectively, assigned to the parameters: α, β, γ and K_s, Eq. 12 becomes: $(48 + 12\varnothing_2^2) \tan \varnothing_2/2 + \varnothing_2^2 (0.5\varnothing_2^2 - 24) \cot \varnothing_2 + 2\varnothing_2^3 = 0$. The first root of this equation exists at $\varnothing_2 = 4.882$, therefore K_1 and K_2 have values of ∞ and 0.6435, respectively. The last couple of values agrees with results available in the literature, the exceptions are the results by Agrawal and Stafiej[2]. Their results indicate values of 0.0 and 0.643 for K_1 and K_2.

It is obvious from the previous examples that the proposed closed-form equations are accurate and applicable regardless of loading and geometric configuration of stepped columns. Also, it is clearly seen that part of the available results need amendment.

7. Summary and Conclusions

Characteristic equations for comprehensive range of end restraints that may exist at the boundaries of stepped columns are presented in this paper. The equations presented account for the elastic restraints that may be imposed at column step. In pursuing the derivation of the models presented, the stiffness formulation of kinematics at shafts step is used; thus, avoiding the lengthy solution of governing differential equations. Regarding the material presented in this paper, the following conclusions are drawn :

1. The presented characteristic equations are accurate as the results obtained compared very satisfactory with the available results.

2. The equations proposed are valid for assessment of effective length factor for : stepped columns, continuous compression chords of plane trusses, and continuous extending through two stories. The latter is popular in steel industry.

3.   The closed-form equations provided in this paper can be computerized with minimal effort.

4. The intermediate spring attached at column step drastically reduces the effective length factor at relatively low rigidity requirement.

5. Experimental work on stability of stepped columns is needed for further substantiation of the existing theoretical models. This is proposed for future research.

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Appendix I

Notation

The following symbols are used in this paper :

C_o Carry over factor;
D Displacement vector;
E Modulus of elasticity;
E_t Tangent modulus of elasticity;
I_1 Moment of inertia of upper shaft;
I_2 Moment of inertia of lower shaft;
K_1 Effective length factor of upper shaft;
K_2 Effective length factor of lower shaft;
K_s Non dimensional stiffness parameter;
L_1 Geometric length of upper shaft;
L_2 Geometric length of lower shaft;
M_2 Moment at near end A due to unit rotation at A;
M_3 Moment at far end B due to unit rotation at A;
P_1 Axial force in upper shaft;
P_2 Axial force in lower shaft;
S Global stiffness matrix;
S_g Geometric stiffness term;
S_{ij} Action at i due to unit displacement at j;
S_s Stiffness of linear spring attached at column step;
V_1 Shear at near end A associated with unit translation at A;
α Inertia of upper to lower shaft, I_1/I_2 ;
β Aspect ratio of upper to lower shaft, L_1/L_2 ;
γ Ratio of axial forces in column shafts, P_1/P_2 ;
$\varnothing_1$ Non-dimensional buckling factor of upper shaft , and
$\varnothing_2$ Non-dimensional buckling factor of lower shaft.

استقـــرار الأعمـــدة المتـدرجـــة

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المستخلص .   لقد تمت دراسـة موضـوع استقرار الأعمـدة المتـدرجـة بالتفصيل . وتم تحـديد العوامل المختلفة التي تؤثر على تصـرف الأعمـدة المتدرجـة تحت تأثير قـوى الضغط الرأسـية وتحليل تأثيـراتهـا وتمثيـلهـا . والعوامل التي تم التطرق إليها هي درجة تقييد الأطراف و المحددات على نقطة تغييـر المقطع ، ونسبة أبعاد مقـاطع الأجزاء المختلفة من العامـود ، ومستوى الأحمال الرأسية المؤثرة على أجزاء العامـود . وفي النهاية تم اقتراح حلول مباشرة لتمثيل استقرار الأعمدة المتـدرجة وتصرفهـا تحت تأثير أحمال الضغط الرأسية مع أمثلة مناسبة .