

# PID Design Using Particle Swarm Optimization and Iterative Linear Matrix Inequality for a Wind Power Plant

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**Abstract** – This paper presents a Proportional-Integral-Derivative (PID) design using two techniques, namely, Particle Swarm Optimization (PIDPSO) and Iterative Linear Matrix Inequalities (PIDILMI), as applied to a multivariable blade pitch control for a wind turbine generating system comprising a synchronous generator connected to a large power system. The PID is used to control the electrical torque delivered to the power system. To show the effectiveness of the proposed techniques, divers tests, namely, step/tracking in the controlled variable, and variation in system parameters, are applied.

## I. INTRODUCTION

Wind energy, a form of renewable source, uses wind turbine to convert the energy contained in flowing air into electricity. The main advantages of electricity generation from renewable sources are the absence of harmful emissions and presumed infinite availability of the prime mover that is converted into electricity [1-4]. Unfortunately, the contribution of wind power is still limited and covers only a small part of the total power system load. The rest is still fulfilled by conventional plants such as thermal, nuclear and hydro power plants. Therefore, the former hardly contributes in voltage and frequency control. Thus, frequency and voltage are controlled by large power plants usually of conventional types. Besides, if a disturbance occurs, the wind turbines are disconnected and reconnected when normal operation has been resumed. The tendency to increase their size will have more influence on overall power system behavior. Thus, their behavior in an electrical power system and interaction with other generating equipments and loads should be looked upon.

A wind turbine is essentially a machine that converts the kinetic energy of the moving air (wind) first into mechanical energy at the turbine shaft and then into electrical energy. The force of the wind creates aerodynamic lift and drag forces on the rotor blades, which, in turn, produce the torque on the wind turbine rotor. All wind turbines are therefore designed with some sort of power control. For the wind turbines, generated power is dependent on the wind speed. However, for large generators, rotor speed changes are quite smooth.

Previous work [5-9] has explored system performance with divers control techniques. In this work, PID setting gains are designed using two techniques. The first uses Particle Swarm Optimization (PSO) [10-13] as an optimization technique for an index representing the error between the reference and actual controlled values. In the second, Iterative Linear Matrix Inequalities (ILMI) technique is used [14-19]. The PID designed using the first controller is named PIDPSO whereas the second is named PIDILMI. In both cases, the PID controller is used to control the electrical torque delivered from a wind turbine generating system comprising a synchronous generator connected to a large power system. The effectiveness of the proposed techniques are demonstrated using divers tests, namely, step/tracking in the controlled variable, variation in system parameters, and gust effect, are carried out. Matlab platform with its control and LMI toolboxes [13,17] are extensively exploited.

## I. SYSTEM MODELING

The wind turbine converts the kinetic energy of the moving air (wind) first into mechanical energy at the turbine shaft and then into electrical energy as depicted in Figure 1.

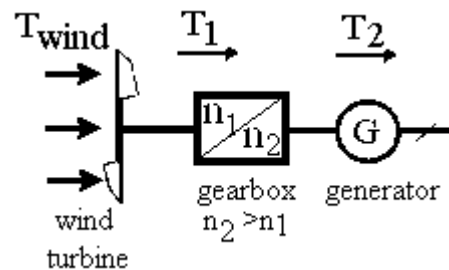


Figure 1. Block diagram of the wind turbine

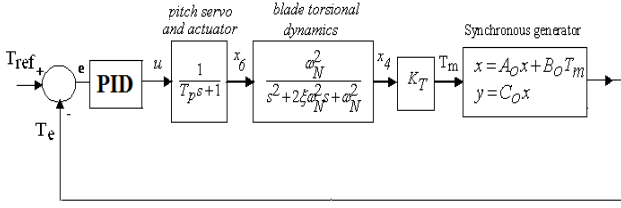


Figure 2. Wind turbine model for the system under study

The block diagram of the system under study is shown in Fig. 2 and the data are given in the appendix.

#### A. State-space representation

The variables shown in Fig. 2 represent small displacements around a selected operating point. The open-loop system can be written as

$$\dot{x} = Ax + B_1 u + B_2 v_g$$

$$y = Cx = T_e$$

$$u = K_p e + K_i \int e dt + K_d \frac{de}{dt} \quad (1)$$

$$e = T_{ref} - T_e$$

Where

$$x = \begin{bmatrix} \delta & \omega & e'_q & T_d & x_5 & x_6 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{\omega_0 k_1}{2h} & 0 & -\frac{\omega_0 k_2}{2h} & \frac{\omega_0 k_{th}}{2h} & 0 & 0 \\ \frac{k_4}{\tau_{do}} & 0 & -\frac{1}{\tau_{do}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau_{do} k_3 & 1 & 0 \\ 0 & 0 & 0 & -\omega_n^2 & -2\xi\omega_n^2 & \omega_n^2 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_p} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{\tau_p} \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 & \frac{\omega_0}{2h} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = [k_1 \quad 0 \quad k_2 \quad 0 \quad 0 \quad 0]$$

And

$u$  control input  
 $v_g$  wind speed or gust

#### I. ITERATIVE LMI PID CONTROLLER

In Fig. 3, a transformation of the PID controller to a Static Output Feedback (SOF) controller is performed

[18,20]. Consider the linear time-invariant system given by (1) and rewritten as follows:

$$\dot{x} = Ax + Bu, \quad y = Cx \quad (2)$$

With  $B=B_1$  and

$$u = F_1 y + F_2 \int_0^t y dt + F_3 \frac{dy}{dt} \quad (3)$$

Where

$x$  state variable  
 $u$  control input  
 $y$  output  
 $A, B$  and  $C$  constant matrices  
 $F_1, F_2, F_3$  matrices to be designed.

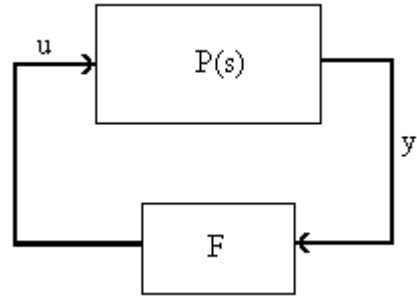


Figure 3 Multivariable PID

Let  $z_1 = x$  and  $z_2 = \int_0^t y dt$ . Denote  $z = [z_1^T \ z_2^T]^T$ . The variable  $z$  can be viewed as the state vector of a new system whose dynamics are governed by

$$\dot{z}_1 = \dot{x} = Az_1 + Bu \quad (4)$$

$$\dot{z}_2 = y = Cz_1$$

i.e.

$$\dot{z} = \bar{A}z + \bar{B}u \quad (5)$$

where

$$\bar{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

Combining (2) and (5) yields

$$y = Cz_1 = [C \quad 0]z$$

$$\int_0^t y dt = z_2 = [0 \quad I]z$$

$$\frac{dy}{dt} = C\dot{x} = CAx + CBu = [CA \quad 0]z + CBu$$

Let

$$\begin{aligned}\bar{C}_1 &= [C \ 0] \\ \bar{C}_2 &= [0 \ I] \\ \bar{C}_3 &= [CA \ 0]\end{aligned}$$

And

$$\bar{y}_i = \bar{C}_i z, i = 1,2,3$$

Thus (3) becomes

$$u = F_1 \bar{y}_1 + F_2 \bar{y}_2 + F_3 \bar{y}_3 + F_3 C B u$$

Suppose  $(I - F_3 C B)$  is invertible, then

$$u = \bar{F}_1 \bar{y}_1 + \bar{F}_2 \bar{y}_2 + \bar{F}_3 \bar{y}_3 \quad (6)$$

Where

$$\begin{aligned}\bar{y} &= [\bar{y}_1^T \ \bar{y}_2^T \ \bar{y}_3^T]^T \\ \bar{F} &= [\bar{F}_1 \ \bar{F}_2 \ \bar{F}_3] \\ \bar{C} &= [\bar{C}_1^T \ \bar{C}_2^T \ \bar{C}_3^T]^T \\ \bar{F}_1 &= (I - F_3 C B)^{-1} F_1 \\ \bar{F}_2 &= (I - F_3 C B)^{-1} F_2 \\ \bar{F}_3 &= (I - F_3 C B)^{-1} F_3\end{aligned}$$

The problem of PID controller design reduces to that of a Static Output Feedback (SOF) controller design for the following system:

$$\begin{aligned}\dot{z} &= \bar{A}z + \bar{B}u \\ \bar{y} &= \bar{C}z \\ u &= \bar{F}\bar{y}\end{aligned} \quad (7)$$

Once  $\bar{F} = [\bar{F}_1 \ \bar{F}_2 \ \bar{F}_3]$  is found, the original PID gains can be recovered from

$$\begin{aligned}F_3 &= \bar{F}_3 (I + C B \bar{F}_3)^{-1} \\ F_2 &= (I - F_3 C B) \bar{F}_2 \\ F_1 &= (I - F_3 C B) \bar{F}_1\end{aligned}$$

The Iterative Linear Matrix Inequalities (ILMI) algorithm for Static Output Feedback (SOF) systems can be found in [19,20]:

## II. SIMULATION RESULTS

To demonstrate the effectiveness of the proposed controllers, several tests are carried out and the results are presented for the wind power plant, described earlier, being driven by each of the proposed controllers. The simulation results are obtained using MATLAB package, control and LMI Toolboxes.

### A. System with PIDPSO

The evolution of the cost function:

$$J = \sum_{t=0}^{t=2} [t^* | T_{ref} - T_e(t) | + 10 * \omega(t)] \quad (8)$$

is shown in Fig. 5. The system is subjected to 10% increase in the reference torque  $T_{ref}$ .

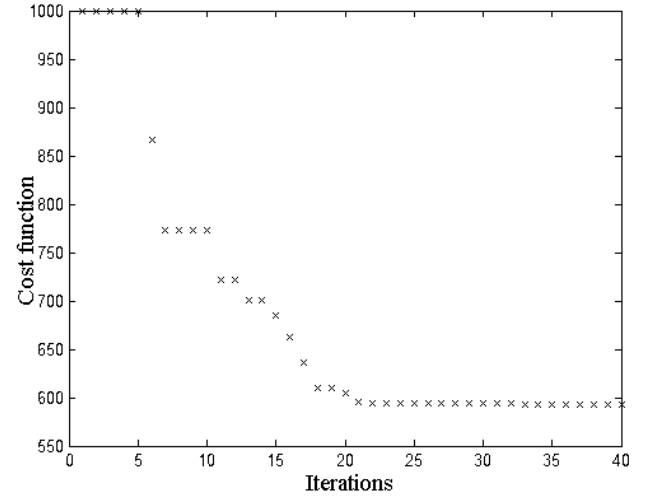


Figure 5 Cost function evolution

The linearized closed-loop state-space system with for PIDPSO is:

$$\begin{aligned}a_{21} &= -\omega_o * k_1 / (2 * h) & a_{23} &= -\omega_o * k_2 / (2 * h) \\ a_{24} &= +\omega_o * k_{th} / (2 * h) & a_{31} &= -k_4 / \tau_{do}' \\ a_{33} &= -1 / (\tau_{do}' * k_3) & a_{54} &= -\omega_n^2 \\ a_{55} &= -2 * \zeta * \omega_n & a_{56} &= +\omega_n^2 \\ a_{61} &= -(K_p * k_1 + K_D * k_2 * a_{31}) / \tau_p & a_{62} &= -K_D * k_1 / \tau_p \\ a_{63} &= -(K_p * k_2 + K_D * k_2 * a_{33}) / \tau_p & a_{66} &= -1 / \tau_p \\ a_{66} &= -1 / \tau_p & a_{67} &= +1 / \tau_p \\ a_{71} &= -K_I * k_1 & a_{73} &= -K_I * k_2 \\ b_{21} &= \omega_o / (2 * h) & b_{22} &= \omega_o / (2 * h) \\ b_{62} &= K_p / \tau_p & b_{63} &= K_D / \tau_p \\ b_{72} &= K_I & & \end{aligned}$$

$$A_{cl} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ a_{21} & 0 & a_{23} & a_{24} & 0 & 0 & 0 \\ a_{31} & 0 & a_{33} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & a_{54} & a_{55} & a_{56} & 0 \\ a_{61} & a_{62} & a_{63} & 0 & 0 & a_{66} & a_{67} \\ a_{71} & 0 & a_{73} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_{cl} = \begin{bmatrix} 0 & 0 & 0 \\ b_{21} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_{62} & b_{63} \\ 0 & b_{72} & 0 \end{bmatrix}$$

PSO data are:

- iterations = 2
- inertia = 1
- correction factor = 2

A set of random numbers for  $K_p$ ,  $K_I$ ,  $K_D$  are generated in [0.01-2], [0.01-1], and [0.01-1] intervals, respectively.

The system closed-loop eigenvalues for PIDPSO are shown in Table 1 while the controller gains are given in Table 2.

**B. System with PIDILMI**

ILMI data are:

The positive definite starting matrix :  $Q=10$   
 The obtained value of  $\alpha$  is:  $\alpha_{opt} = -5.4091$  new

The system closed-loop eigenvalues for PIDILMI are shown in Table 1 while the controller gains are given in Table 2.

TABLE 1.

CLOSED-LOOP EIGENVALUES FOR THE SYSTEM WITH EACH OF THE PROPOSED CONTROLLERS

Eigenvalues	
PIDPSO	PIDILMI
-373.17	$-0.98516 \pm 91.158i$
-34.516	$-6.8155 \pm 40.503i$
$-3.6113 \pm 10.669i$	-7.2102
$-2.7672 \pm 3.505i$	-3.6468
-2.7246	-0.7099

TABLE 2.

CLOSED-LOOP PID GAINS FOR THE SYSTEM WITH EACH OF THE PROPOSED CONTROLLERS

PID Feedback gains	
PSO	ILMI
0.052205	1.298
0.30318	0.1719
0.019398	0.89351

**Test 1: Tracking-response**

To test the effectiveness of the system to tracking the reference value of the torque, the system is subjected to a variation in  $T_{ref}$  as shown in Fig. 6 (a). The system response for the electromagnetic torque  $T_e$ , control input  $u$ , generator speed  $\omega$ , and power angle  $\delta$  are shown in Fig. 6. The system, equipped with the proposed controllers, is subjected to an increase by 10% then a decrease by 10% in  $T_{ref}$ . PIDILMI shows smooth response as compared to an oscillatory response but with relatively larger overshoots exhibited by PIDPSO with heavy oscillations.

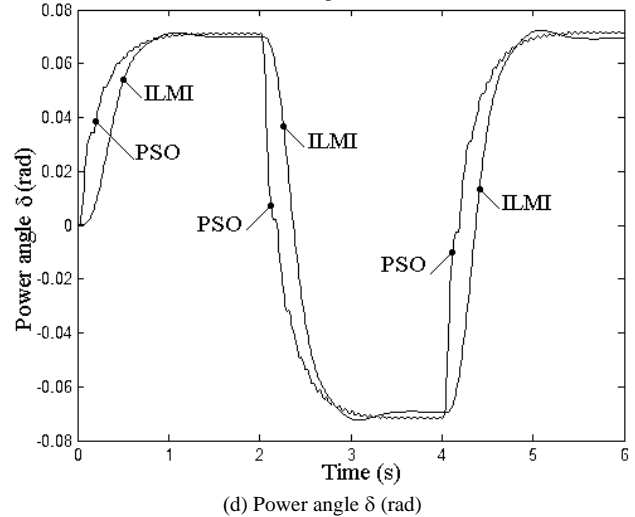
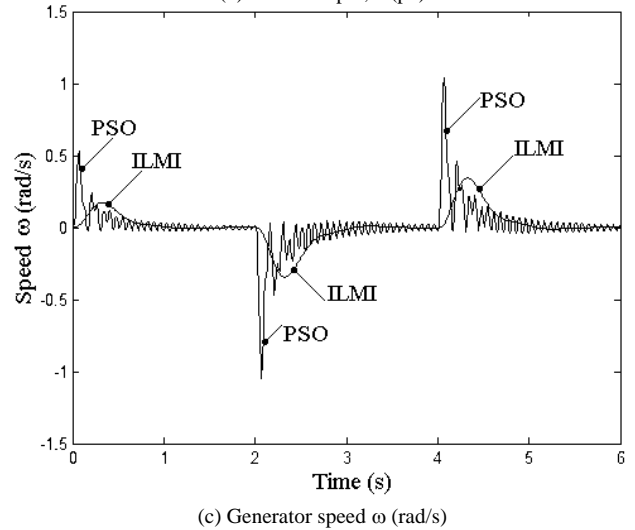
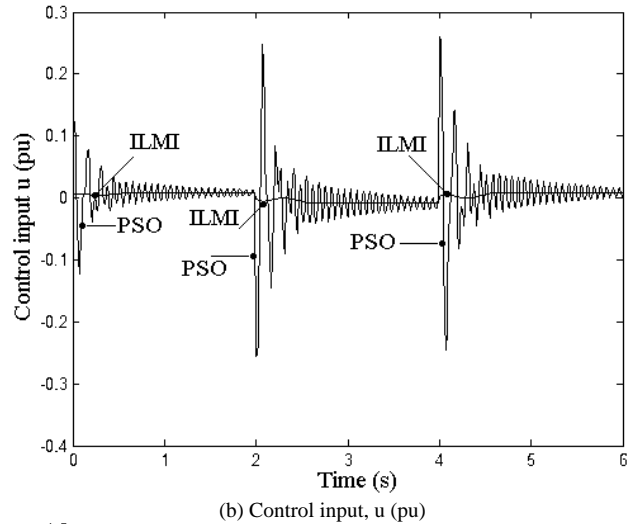
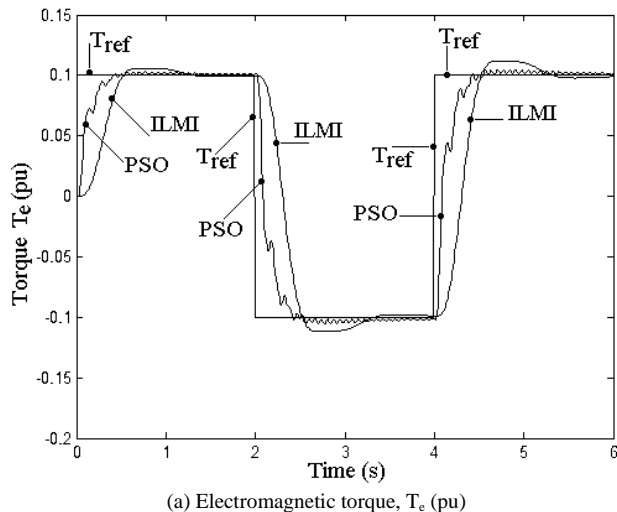
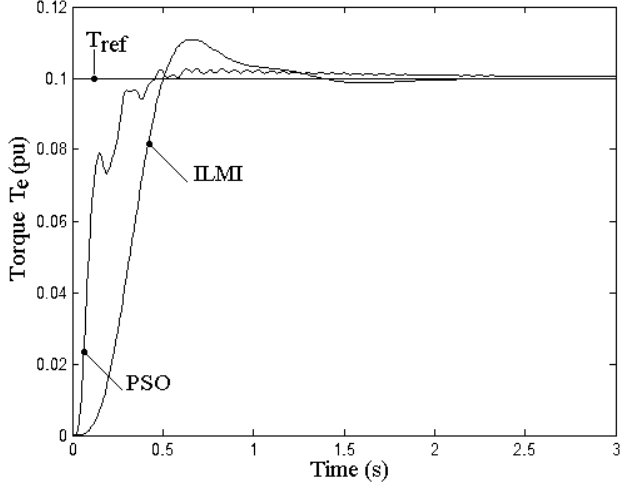


Figure 6. Step-response following  $T_{ref} \pm 5\%$  the -5%

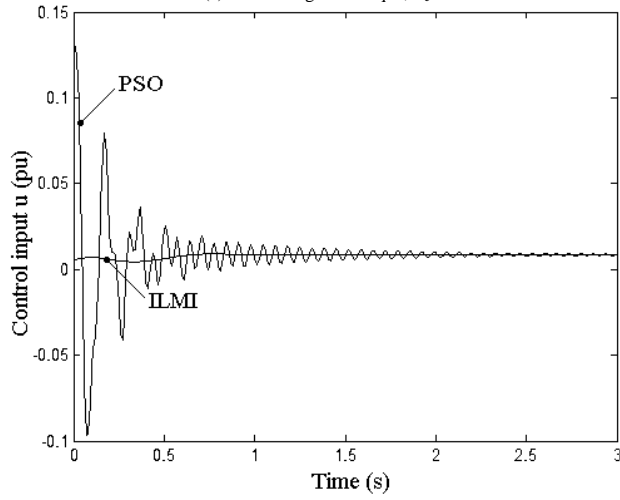
**Test 2: Parameters Variation**

To test the robustness to parameters change, a 50% increase in the field transient time constant  $\tau_{do}$ , 20% increase in the inertia constant  $h$ , and 20% increase in the armature resistance  $r_a$ . Fig. 7 shows the system response following a change in  $T_{ref}$  by +10% while experiencing the described parameters change. It is worth noting that the PID gains used are those found with nominal system

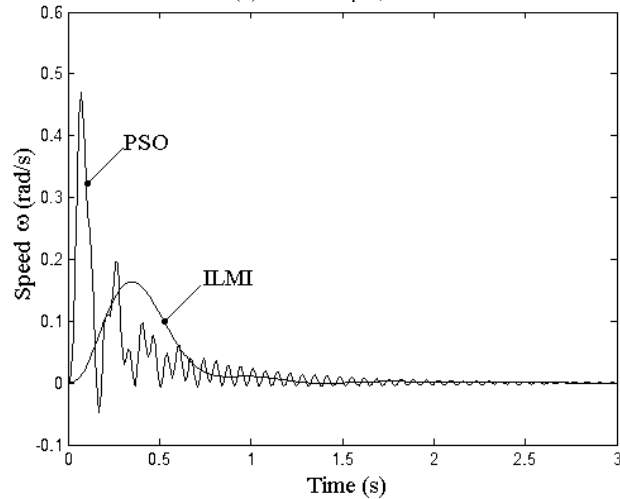
parameters. It is clear that the system responds smoothly with ILMI but slower than PSO where pronounced oscillations are clearly shown.



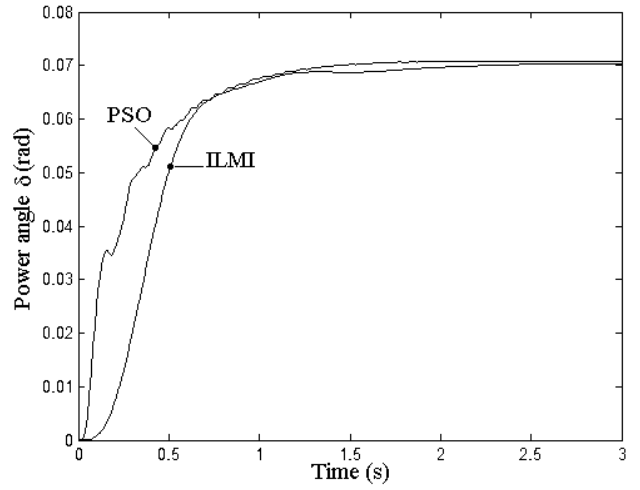
(a) Electromagnetic torque,  $T_e$



(b) Control input,  $u$



(c) Generator speed  $\omega$

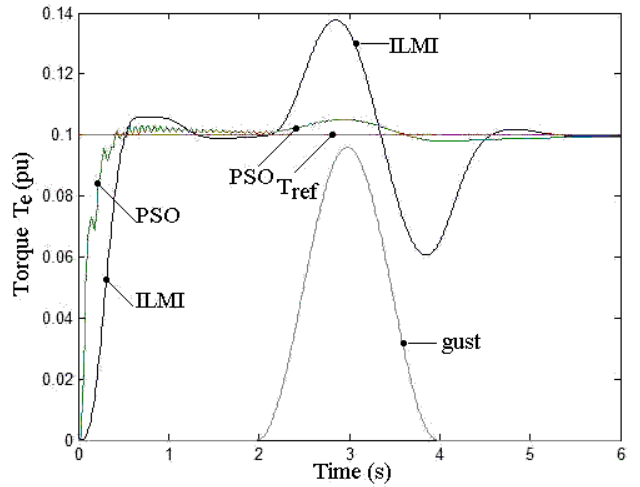


(d) Power angle  $\delta$

Figure 7. Parameters ( $\tau_{d0}'$ ,  $h$ ,  $r_a$ ) variation

**Test 3: Wind Speed Variation (Gust) effect**

Wind gust is a sudden and brief increase in wind speed. To test the robustness to the wind speed variation, a gust shown in Fig. 8(a) is applied between 2-4 seconds. Fig. 8 shows the above mentioned variables behavior following a change in  $T_{ref}$  by +10% from  $t=0$  s. It is clear that ILMI shows smooth with higher overshoot as compared to PSO that shows faster response but with heavy oscillations. Both controllers exhibit stability following the appearance of the gust.



(a) Electromagnetic torque,  $T_e$

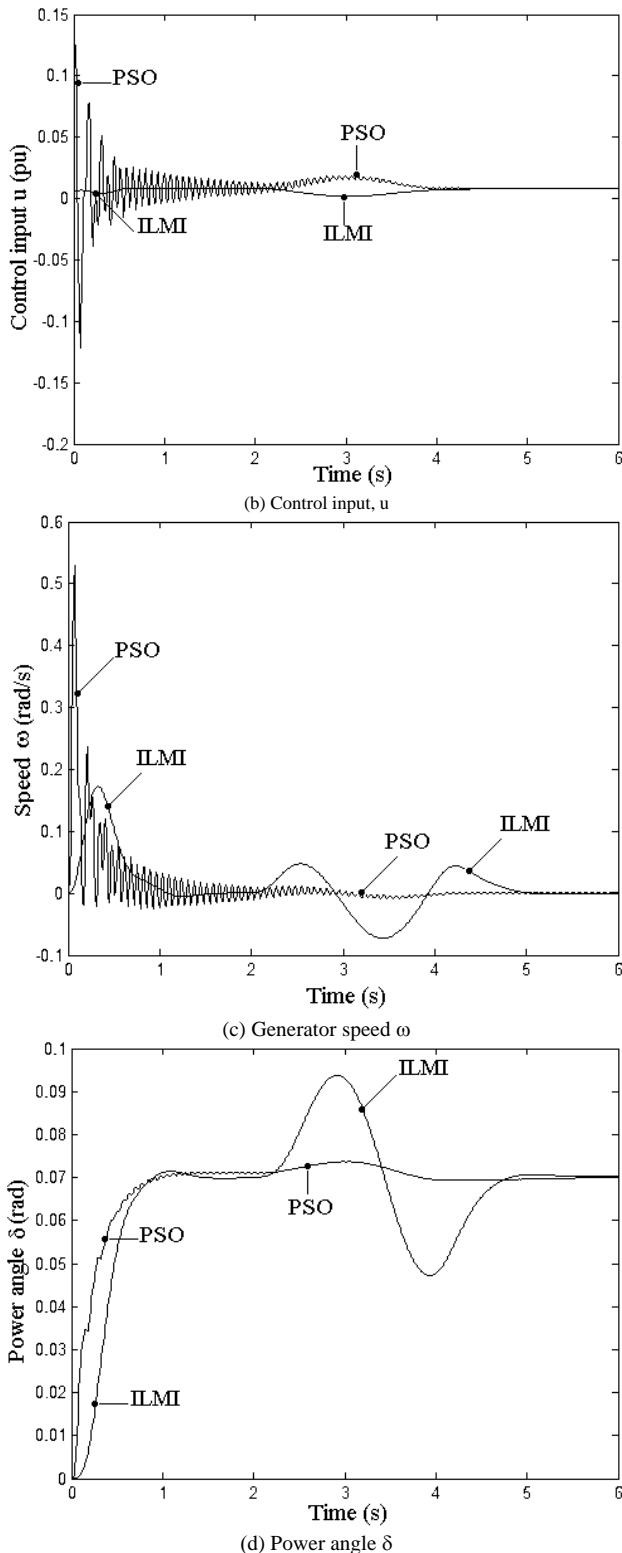


Figure 8. Gust effect on system response

## II. CONCLUSION

A PID controller is designed using two techniques, named, Particle Swarm Optimization (PSO) and Iterative Linear Matrix Inequalities (ILMI), and applied separately to drive a power system comprising a wind turbine and a synchronous generator connected to an infinite bus via a step-up transformer and a transmission line. Both designed controllers exhibit stability to the applied tests.

The ILMI shows smoother but slower response whereas PSO shows faster one with heavy oscillations that need to be absorbed by the use of, for example, a Power System Stabilizer (PSS).

The advantages of one method with respect to the other depend on the designer needs and constraints. The system design is done for one operating point, it is worth looking into adapting the controllers to different operating points.

## REFERENCES

- [1] H. H Hwang, Leonard J. Gilbert, "Synchronization of wind turbine generators against an infinite bus under gusting wind conditions," *IEEE Trans. on Power Apparatus and Systems*, Vol. PAS-97, no. 2, March/April 1978, pp. 536-544.
- [2] M T Iqbal, A H Coonick, L L Freris, "Some control aspects of a wind turbine," *IEE*, 1993, pp. 6/1-6/3.
- [3] J.G. Slootweg W.L. Kling, "Modeling of Large Wind Farms in Power System Simulations", 0-7803-7519-X/02/ © 2002, *IEEE*, pp. 503-508
- [4] A. D.Anca D. Hansen, P.Poul Sorensen, L.L. Janosi, and J. Bech, "Wind farm modeling for power quality," *Proc. IEEE IECON'01*, vol. 3, 2001, pp. 1959-1964.
- [5] Stol, K., Zhao, W., & Wright, A., 2006, "Individual Blade Pitch Control for the Controls Advanced Research Turbine (CART)", *ASME J. Solar Energy Engineering*, vol. 128(4), pp 498-505..
- [6] L. Mihet-Popa, F. Blaabjerg, and Ion Boldea, "Wind Turbine Generator Modeling and Simulation Where Rotational Speed is the Controlled Variable," *IEEE trans. On Ind. Appl.*, Vol. 40, No. 1, Jan/Feb 2004, pp. 3-10.
- [7] M. E. Bahgat, A. M. Abdel Ghany, O. H. Abdalla, "Multivariable controller for a wind driven generator," *Proc. 4<sup>th</sup> Afro-Asian Intern. Conf.*, 12-15 December 1998, Cairo - Egypt.
- [8] S.H. Javid, A.M., and J. R. Winkelman, "Control design for a wind turbine-generator using output feedback", *IEEE Paper 0272-1 708/82/0900-00230*, Sept. 1982.
- [9] A. Bensenouci, A.M. Abdel Ghany, A.M.A. Alzahrani, "Proportional-Integral State-Feedback Control for Wind Power Plant," *Proc. 2<sup>nd</sup> International Conference on Electrical and Electronics Engineering, ICEEE'08*, Laghouat, Algeria, 21-23 April 2008.
- [10] Kennedy J. and Eberhart R., "Particle Optimization," *IEEE: International Conference on Networks*, vol. 4, pp. 1942 - 1948,1995.
- [11] Eberhat and Yuhui Shi, "Particle Swarm Optimization: developments, applications, and resources," *Proc. of the 2001 Congress on Evolutionary Computation*, vol. I, pp. 81-86, 2001.
- [12] N.A. Al-Nusabi, "Variable structure load frequency controller using particle swarm optimization technique", 0-7803-8163-7/03/2003 *IEEE*, pp. 380383.
- [13] B. Birge, " Particle swarm optimization toolbox," [www.mathworks.com/matlabcentral/](http://www.mathworks.com/matlabcentral/).
- [14] C. Scherer, P. Gahinet, and M. Chilali, "Multi-objective output-feedback control via LMI optimization," *IEEE Trans. Automat. Contr.*, vol. 42, pp. 896-911, 1997.
- [15] A.M. Abdel Ghany , "Power System Automatic Voltage Regulator Design Based on Static Output Feedback PID Using Iterative Linear Matrix Inequality", *12<sup>th</sup> International Middle East Power Systems Conference, MEPCON'2008*, South Valley University, Egypt, March 11-13, 2008, pp. 441-446.
- [16] M. Chilali and P. Gahinet, " $H_\infty$  design with pole placement constraints: An LMI approach," *IEEE Trans. Automat. Contr.*, vol. 41, no. 3, March 96, pp. 358-67.
- [17] P. Gahinet, A. Nemirovski, A.J. Laub, M. Chilali, *LMI Control Toolbox for Use with MATLAB*, The MathWorks User's Guide, Version 1, May 1995.
- [18] Y.Y. Cao, J. Lam, and Y.X. Sun, "Static Output Feedback Stabilization: an LMI approach," *Automatica* 34 (1998) 1641-45.
- [19] Ahmed Bensenouci, Iterative-LMI Multivariable PID and Robust LMI Output Feedback  $H_\infty$  Controllers for an Electric Power Plant, *Proc. 4<sup>th</sup> International Conference on Computer Integrated (CIP'2007)*, 03-04 November 2007, Setif, Algeria.

- [20] Feng Zheng, Qing-Guo Wang, and Tong Heng Lee, "On the design of multivariable PID controllers via LMI approach," *Automatica* 38 (2002) 517–526.



APPENDIX

TABLE III.  
OPERATING CONDITIONS

infinite bus voltage	$V_a=1$
reactive power	$Q=0.6$
active power	$P=0.8$
gear ratio	$N=37.5$
Torque factor:	$k_{th}=11.86$
System constants:	$k_1=2.492441223$
	$k_2=2.514312798$
	$k_3=0.082959641$
	$k_4=5.141769761$

TABLE IV.  
SYSTEM DATA

transmission line resistance	$r_e=0$
transmission line reactance	$x_e=0.02$
r.p.m	$n_r=40$
lade radius	$r_b=62.5$
wind speed m/sec	$v_p=18$
no. of poles	$pp=4$
inertia constant	$h=9.5$
conventional integral power controller gain	$k_p=0.075$
zeta	$\zeta=0.02$
exciter time constant	$\tau=0.05$
exciter gain	$k_e=30$
P.F controller gain	$k_{pf}=0.2$
generator armature resistance	$r_a=0.018$
d-axis reactance	$x_d=2.21$
q-axis reactance	$x_q=1.064$
transient d-axis reactance	$x_d'=0.165$
subtransient d-axis reactance	$x_d''=0.128$
subtransient q-axis reactance	$x_q''=0.193$
d-axis transient field time constant	$\tau_{do}'=1.94212$
d-axis subtransient field time constant	$\tau_{do}''=0.01096$
q-axis subtransient field time constant	$\tau_{qo}''=0.0623$
angular speed of the generator [rad/sec]	$\omega_o=100\pi$
$\omega_n$	$\omega_n=100$
wind turbine filter time constant	$\tau_p=1/(2*2.7*\pi)$
Torque factor	$k_{th}=11.86$

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He got promoted to Associate Professor in Ecole Nationale Polytechnique, ALGERIA in 1992 where he was working since 1989. From 1994 to 1996, he worked at the University of 7<sup>th</sup> of April, Zawia, Libya and during 1996 to 1998, at the University of Malaya, KL, Malaysia, as a Staff member. From 1998 until 8,2008, he is with the College of Technology at Al-Baha, Saudi Arabia. His research areas include Power System Modeling and Analysis and Application of Advanced Techniques (Neural Networks, Fuzzy, Genetic Algorithm, Variable Structure, ...etc) and Robust Control techniques to the Analysis and Control of Power System, electric machines and drive systems.

Dr. Benseouci was awarded distinguished lecturer of the academic year 2000/2001. He published more than 90 papers in divers local and international Conferences and Technical Journals. He was the general secretary of the 1<sup>st</sup> Baha Technical Meeting, BTM'2004, Baha, KSA, May 3-5, 2004.