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Delineation of Basement Surface Relief from its Magnetic Anomaly Using Hybrid Numerical Algorithm

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Abstract. Magnetic field mapping is a very powerful geophysical tool for recognizing basement relief. In the present work, the problem of finding the depth to the basement surface is transformed into the problem of minimizing a constraint objective function $\psi(z_j)$ of the L_1 norm instead of the traditional L_2 in the unknown depths (z_j). The assumed subsurface model is consisting of a number of vertical prisms, homogeneously magnetized and of fixed magnetic susceptibility κ. The Finite Difference-Quasi Newton (FD-Newton) algorithm with active set strategy (ASS) is used as a hybrid technique to minimize the constraint objective function $\psi(z_j)$. Both linear and nonlinear bound constraints are implemented as a priori information to reduce ambiguity and improve the solution. For simplicity, the problem can be solved as an even determined; however, it is directly solved as an over-determined with little modification The method is found to be stable, robust and convergent even for large size models and when implementing inherent noise in the synthetic examples. Further, the present method is applied to a field example from the gulf of Suez, Egypt. The calculated depths show a good agreement with results of drilled wells in that area.

Introduction

Parameter estimation is a common target in processing geophysical data both in linear and non-linear problems. The formulation of such problems may be classified into two main categories. The first category of formulation is dealing with the source geometry, while fixing its physical property (density, magnetic susceptibility, thermal conductivity, etc). The other formulation is dealing with physical property distribution in a geometrically and structurally fixed domain.

Many authors work in the first category (Al-Chalabi, 1972; Parker, 1974; Sabatier, 1977a; Pedersen, 1977,1979; Gupta, 1983; Granser 1987; Chai and Hinze, 1988; Abd El-Azeem, 1993, Gobashy, 1993 & Barbosa *et al*., 1999b). Many authors have also investigated the second class of formulation for optimization and parameter estimation. Mottl and Mottlova (1972) and Safon *et al.* (1977) used the integer linear programming. Fisher and Howard (1980) used the linearly constrained least-squares method and the quadratic programming. Last and Kubik (1983) introduced the minimum volume concept (maximum compactness). Manichetti and Guillen (1983) used the same concept of minimum volume but the technique is broadened to include the search for solutions minimizing the moment of inertia with respect to the center of gravity or with respect to a given dip line passing through it. The resulting structures are both deeper and more compact, precisely as is required in specific cases. Pilkington (1997) used conjugate gradient as an optimization tool for 3-D imaging of magnetic data. Barbosa *et al.* (1999a) proposed a technique for delineating the basement relief by determining the physical properties (densities in his case) of the 2-D array of rectangular prisms by using non-smoothness constraint on the interface to be estimated. Abdel Azeem (2001) used conjugate gradient and FD-Newton for inverting such 2-D array of prisms.

Pedersen (1977) proposed a 2-D earth model of rectangular prisms extending to infinity in the y-direction and used the generalized matrix inversion approach for parameter estimation. He assumed a single susceptibility (or density) contrast to all prisms and calculated the depth to top from magnetic (or gravity) data. In this paper, a new hybrid inversion technique is used for inverting surface magnetic data; to recover depth to basement surface using Finite-Difference for estimating the gradient together with modified (Quasi)-Newton method with active set strategy (ASS) to reduce the ambiguity and improve the solution. To mimic reality, a 2-D uniformly magnetized earth model consisting of vertical prisms with constant magnetic susceptibility contrast is used.

Formulation of the Problem

To solve the magnetic inverse problem numerically; the basement is subdivided into a raw of rectangular prisms (m) with constant magnetic susceptibility contrast and infinitely extended in the y-direction (Fig. 1). The magnetic effect at the ith data point can be given as (Grant and West, 1965 & Pedersen, 1977):

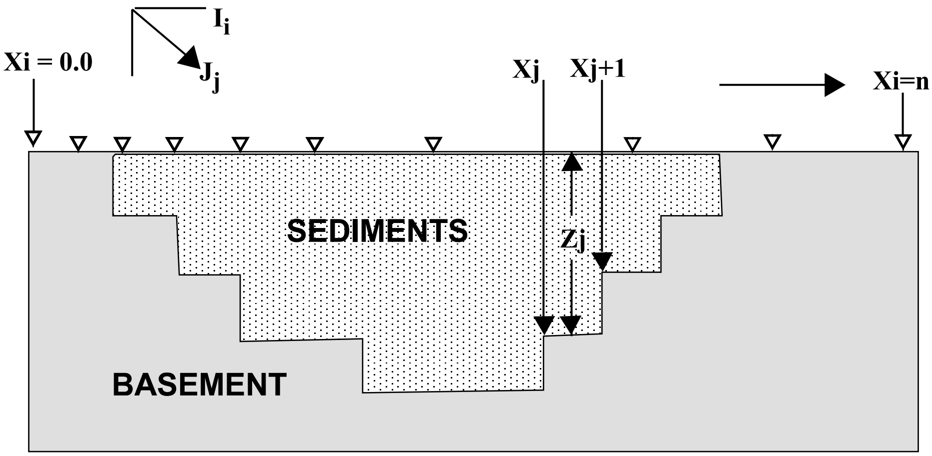

FIG. 1. Two-dimensional earth model.

$$\Delta T(x_i) = 2\Delta\kappa H_o(1-\cos^2 I\cos^2\lambda)\sum_{j=1}^{m-1}\left\{\begin{array}{l}\cos 2\beta(-\tan^{-1}\dfrac{x_{j+1}-x_i}{z_j} + \tan^{-1}\dfrac{x_j-x_i}{z_j}) \\ +\dfrac{1}{2}\sin 2\beta \ln\left(\dfrac{(x_{j+1}-x_i)^2+z_j^2}{(x_j-x_i)^2+z_j^2}\right)\end{array}\right\}$$

$$+2\Delta\kappa H_o(1-\cos^2 I\cos^2\lambda)\sum_{j=1,m} S_i\left\{\begin{array}{l}\cos 2\beta \tan^{-1}\dfrac{x_j-x_i}{z_j} \\ -\dfrac{1}{2}\sin 2\beta \ln((x_j-x_i)^2+z_j^2)\end{array}\right\} \quad (1)$$

where, $\Delta\kappa$: is the susceptibility contrast,

I : is the inclination angle,

λ : is the declination angle,

H_o : is the ambient field,

$\tan^{-1}\beta =$ (tan I / sin λ), and

$S_i = \{-1\ ; j = 1$, and $= 1\ ; j = m\}$.

To formulate the problem, an objective or fitness function $\psi(z_j)$ is introduced. It displays the error between the observed and calculated effects. The unknown parameters (z_j) in the present case will be the depths to the basement surface resembled by the upper face of each prism (j).

Unlike the gravity problem, the magnetic problem treated in this work is less linear. Tense, a robust and more stable form of the objective function may be expressed in its L_1 norm instead of L_2 one (Gobashy, 1993, 2000), this is given as :

$$\psi(z_j) = \sum_{i=1}^{m} \left| \Delta T_{obs}(z_i) - \Delta T_{calc}(z_i) \right| \quad (2)$$

where, ΔT_{obs} is the observed field, and

ΔT_{calc} is the calculated one.

In comparison with the L_1 norm, usage of least-squares formulation to minimize this type of non-linear functions tends to be very sensitive with errors (Bube and Langan, 1997). This is common in real data and implementation of L_2 norm implies also *highly* overdetermined problem ($n >> m$; $n \approx 20*m$), (Dennis and Schnabel, 1996), which is not easily available in real cases. The target now is to minimize $\psi(z_j)$ subject to equality and inequality constraints to estimate the unknown depths z_j using a robust hybrid approach to minimize this non-linearity, *i.e.*, in a mathematical form;

$$\min_{z \subseteq R^m} \psi(z_j), \quad l \leq z_j \leq u \quad (3)$$

where l and u are the lower and upper bounds respectively.

The Algorithm

To achieve robustness, a mixed form of modified (quasi)-Newton method and finite difference Hessian approximation together with an active set strategy ASS (Dennis and Schnable, 1996) is used to solve the minimization form stated in equation (3), subject to bounds on the variables (depths). In an algorithmic scene, the problem is stated as follows: From a given starting point z_c, an active set IA, which contains the indices of the variables at their bounds, is built. A variable is called a "free variable" if it is not in the active set. The search direction for the free variables is then calculated according to the formula;

$$d = -B^{-1} g_c, \quad (4)$$

where, B is a positive definite approximation of the Hessian, and g_c is the gradient evaluated at z_c; both are computed with respect to the free variables.

The search direction for the variables in IA is set to zero. A line search is used to find a new point z_n, $z_n = z_c + \gamma d$, $\gamma \in (0,1]$, such that $\psi(z_n) \leq \psi(z_c) + \alpha g^T d$, $\alpha \varepsilon (0,0.5)$, where g^T is the transpose of matrix g.

To get a stable and geologically accepted solution in potential field interpretation, the inversion method must consider particular constraints. These constraints will inevitably restrict the type of geological setting where the method may be applied. There are many types of constraints such as, lower and upper bounds of parameter estimates, proximity of parameter estimate to a specified value, proximity between pair of parameter estimates, concentration of the anomalous source about a geometrical element such as axis, ... *etc.* (Silva *et al.*, 2000). These constraints have been applied by Sabatier (1977a), Pilkington and Crossley (1986), Barbosa *et al.* (1997), Barbosa *et al.* (1999b), Camacho *et al.* (2000) and many others. The constraints used in the present work are equality and inequality constraints estimated from available geophysical data and geological background.

The optimality conditions, or the stopping criterion used is:

$$\| g(z_j) \| \leq \varepsilon,\ l_j < z_j < u_j,\ g(z_j) < 0,\ z_j = u_j \text{, and } g(z_j) > 0,\ z_j = l_j .$$

These conditions are checked, at each iteration, where ε is a gradient tolerance. When optimality is not achieved, B (the positive definite approximation to the Hessian) is updated according to the BFGS (positive definite secant update) formula (Dennis and Schnabel, 1996):

$$B \leftarrow B - \frac{B_{ss}^{T}\ B}{s^{T}\ B_{s}} + \frac{yy^{T}}{y^{T}\ s} \tag{5}$$

where, $s = z_n - z_c$ and $y = g_n - g_c$

This form is found to be effective in forcing the algorithm to converge. Another search direction is then computed to begin the next iteration until optimality condition is met. As explained before, the bound constraints are introduced through the concept of active set strategy ASS (Gill and Murray, 1976). The active set, named AI, is changed only when a free variable hits its bounds during iteration or the optimality condition is met for the free variables, but not for all variables in IA, the active set. In the latter case, a variable that violates the optimality condition will be dropped out of IA. Figure 2 shows the general flow of the proposed hybrid algorithm and its structure.

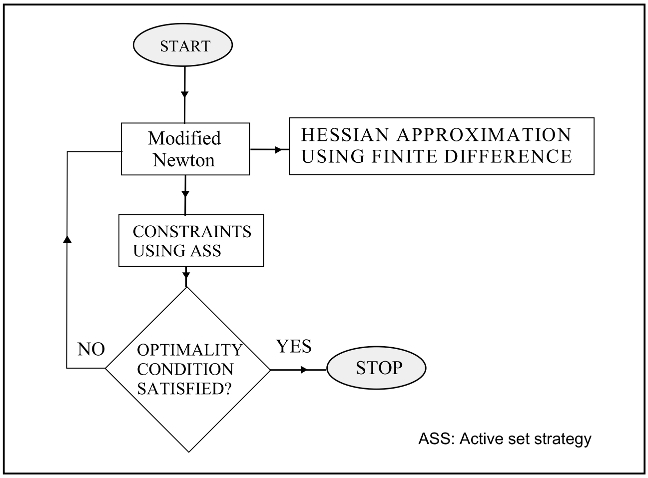

Fig. 2. Structure of the proposed hybrid algorithm.

Application to Synthetic Data

To test the efficiency of the present inversion technique, it is applied on two synthetic subsurface earth models. The first model consists of 24 vertical prisms of 13 km width for each (Fig. 3). Geologically, it represents numerous faulted blocks with central graben and horst structures. The susceptibility contrast between the basement and the overlying sediments is assumed to be 0.002 cgs units. The simplest approach to solve the inverse problem of this type is to assume an even problem. An accepted solution was achieved with this setting. However, due to the non-linearity nature of the problem, increasing the data will transfer it to an over-determined one. Figure 3 gives the solved even-determined problem, while Figure 4 shows the over-determined solution. In both cases, the bound constraints on parameters (depths to top of prisms) are fixed for all prisms from 1.5 to 10 km and an initial guess is given to start up the iterations. The drilling information may be introduced to reduce the ambiguity and decrease the number of iterations required.

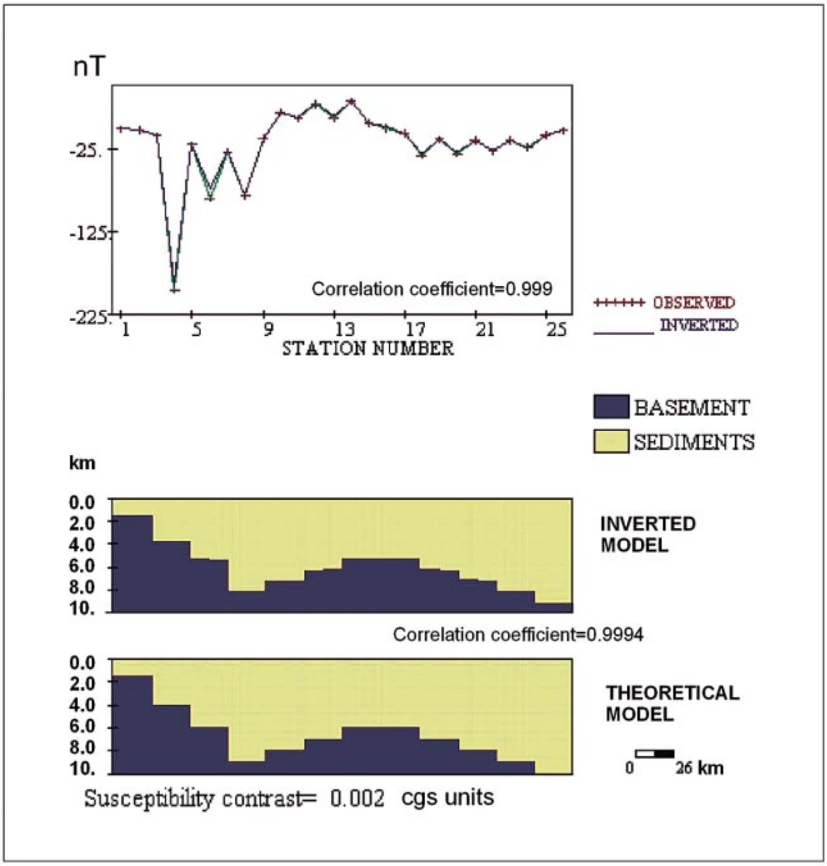

FIG. 3. Inversion results using finite difference-Newton method with active set strategy for 24 prisms, noise free, even-determined problem.

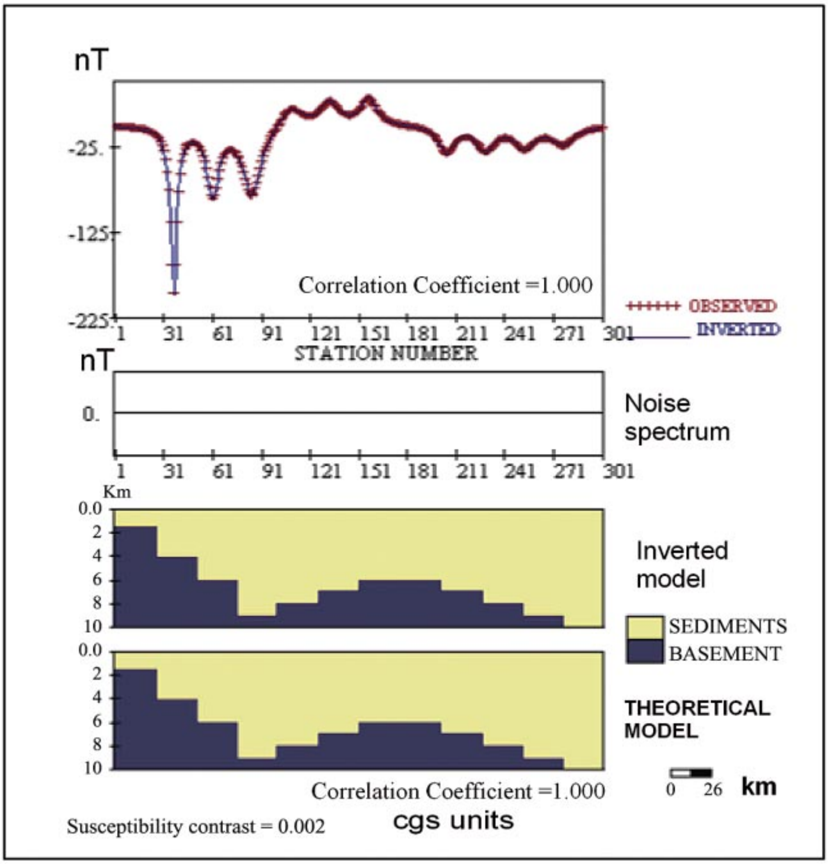

FIG. 4. Inversion results using finite difference-Newton method with active set strategy for 24 prisms, noise free, over-determined problem.

Application of the hybrid technique proposed in this work proves its efficiency in all synthetic test cases introduced. The estimated parameters in all cases are close to true ones. To ensure numerical stability and to simulate artificial errors in the measurements, a frequency-independent random distribution of values (white noise) is added to the observed field .The noise is added to the data with different upper limit percentages (5, 10, 15, 20 and 25% up to 40%) from the field values. Figure 5 displays an example of a solution with 40% noisy data for an overdetermined problem. Table 1 shows the estimated depths in some cases for comparison. In some of these cases (anomaly mixed with noise), very minor misfit between the observed and calculated anomaly profiles can be observed. This is due to the complicated nature of the example and do not affect the inverted model as seen in (Fig. 5). No more fitting can be achieved, where the optimality condition is met automatically as mentioned before.

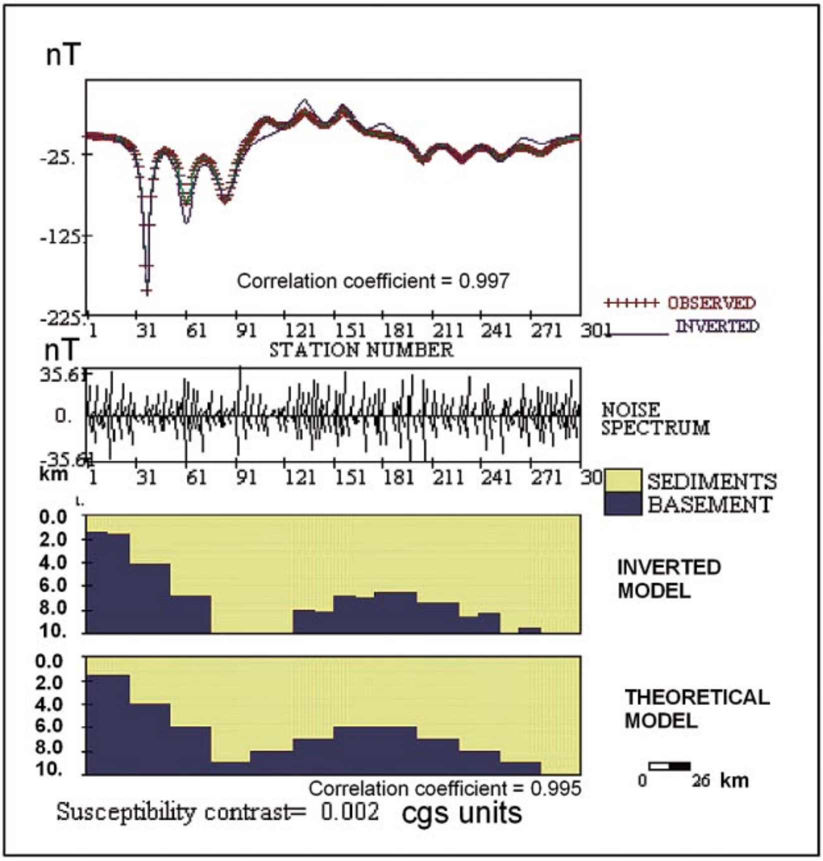

FIG. 5. Inversion results using finite difference-Newton method with active set strategy for 24 prisms, with 40% white noise added to the observed field (over-determined solution).

TABLE 1. Inverted depths as obtained from different noisy fields, where the percentages of the original noise in the observed fields are shown.

Prism no.	True depth (km)	Recovered depths in (km) from different noisy fields			
		10%	20%	30%	40%
1	1.56	1.5	1.5	1.5	1.5
2	1.56	1.5	1.5	1.5	1.55
3	4.06	4.25	4.10	3.97	4.12
4	4.06	4.29	4.14	3.97	4.08
5	6	6.57	6.63	6.40	6.82
6	6	6.52	6.77	6.46	6.83
7	9	10	10	9.29	10
8	9	10	10	10	10
9	8	8.53	10	8.51	10
10	8	10	8.37	8.46	10
11	7	7.08	7.63	7.59	7.99
12	7	7.53	7.65	7.83	8.23
13	6	6.40	6.47	6.64	6.88
14	6	6.41	6.48	6.63	6.99
15	6	6.32	6.24	6.45	6.50
16	6	6.25	6.28	6.47	6.58
17	7	7.27	7.25	7.49	7.47
18	7	7.25	7.24	7.50	7.49
19	8	8.17	8.13	8.57	8.60
20	8	8.16	8.16	8.61	8.41
21	9	9.03	9.08	10	10
22	9	9.12	9.16	10	9.48
23	10	10	10	10	10
24	10	10	10	10	10

In all studied cases, the basement configuration is recovered successfully. It is evident that the hybrid finite difference-Newton with active set strategy is very powerful in estimating unknown parameters even with large sized problems.

The second test example represents one side of a basin-like structure. It consists of 14 vertical prisms, homogeneously magnetized and of inclination 45 degrees and declination 90 degrees. The ambient field H_0 is considered as 50000 nT. The true field is mixed with 40% noise and the solution is shown in Fig. 6. The inverted model provides an excellent agreement with the true model at shallow depths (< ≈ 4 km) even when there is 40 % noise in the original data and then it starts to provide an overdeterminent depth. However, the overdeterminent solution is generated because of the fact that the original data are highly noised. The correlation coefficient between the true and estimated depths is 0.9978. Generally, testing algorithm with extreme noise level such as 40% provides good agreements with true models. Thus, gaining the true model from highly noised magnetic data using the present algorithm can be achieved.

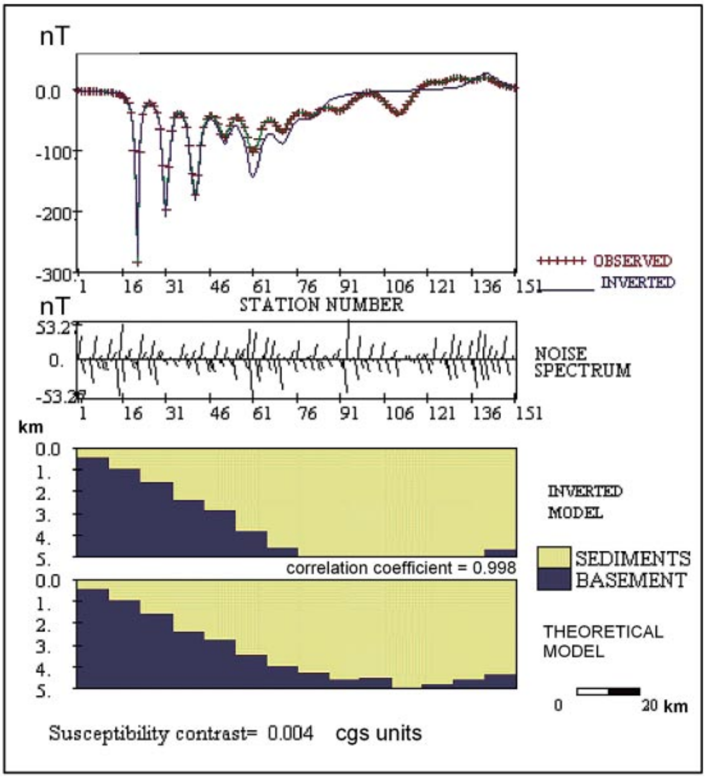

FIG. 6. Solution of inverse problem of a basin-like structure as an even-determined problem. The original data are mixed with 40% noise. Correlation coefficient between true and estimated depths is 0.9987.

Application to Real Data

The present technique is applied to a real field example from the Ras Gharib area, Gulf of Suez, Egypt. Economically, the area is characterized by its rich content of hydrocarbons and oil production. Meshref *et al*. (1976) summarized the tectonics of this area and noted that it lies within the second (central) tectonic province between Ras Zafarana and Ras Shukheir areas in the Gulf of Suez region. Khattab and Hadidi (1961) described the area as a huge graben with successive block faulting away from its center and parallel to it. The main graben had been formed in Pre-Miocene times after a long period of uplifting and erosion as well as igneous activity that clasped in Post-Eocene times (most probably during Oligocene). Therefore, the Miocene Sea had transgressed over Pre-Miocene formations with a major erosional unconformity. Consequently, the Miocene sediments in the Gulf area are found to be resting on different formations ranging from Basement to Eocene. The average inclination in the studied area is 41 degrees, and the declination is 2 degrees (Mekkawi, 1998). A N-S profile (AA') is digitized to a 55 stations from the original total intensity aeromagnetic anomaly map as shown in Fig. 7(a) (EGPC, 1983) over a well-defined magnetic high anomaly. The prism width is taken to be 0.43 km. The earth model was assumed to consist of 50 prisms homogeneously magnetized each with constant magnetic susceptibility contrast (0.00297 SI units). Few constraints are implemented from the available geophysical and geological information. The finite difference- (Quasi) Newton with active set strategy is applied on the profile to estimate depths to basement surface. The results are shown in Fig. 7(b). A recovered basement relief (mathematical model) is shown. The average depth to basement surface is 2960 m. Available drilling information from the close wells shows that the depths to basement are as follows: Jetti-1x, 3200 m; S.El Yusr-1, 2800 m; El Yusr-1, 2850 m. The results of the inverted model (Fig. 7b) confirm well these depths.

Conclusion

The application of hybrid numerical algorithms in parameter estimation, especially in large size problems, is a powerful approach. The modified quasi-Newton algorithm, using the Finite difference derived Hessian and controlling the constraints either linear or non-linear with the active set strategy, minimizes the objective function with the desired tolerance. Application of the proposed technique is recommended on highly rugged basement surfaces or tectonically complex regions assuming homogeneous non-magnetic overlying sedimentary cover.

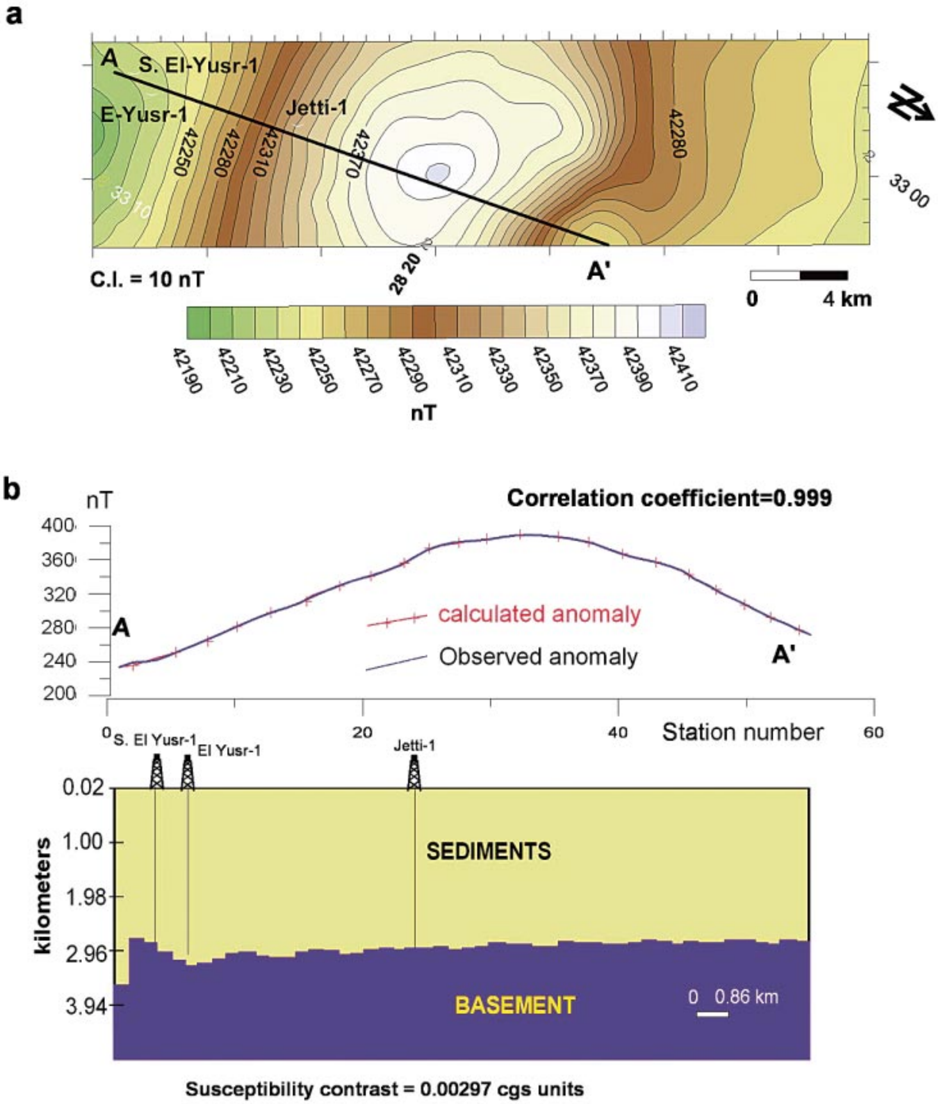

FIG. 7(a). Total intensity aeromagnetic anomaly map of Ras-Gharib area (EGPC, 1983). Illustrated on the map, the profile under study AA' and the controlling wells.
(b). Solution of the over-determined inversion problem along profile AA'.

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تحديد سطح صخور القاعدة من الشا ذات المغنطيسية باستخدام خوارزم عددى مهجن

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المستخلص . يعتبر التمثيل ذوالبعدين للمجال المغنطيسي أداة جيوفيزيقية فعالةً لاستنباط تضاريس سطح الصخور القاعدية . في هذا البحث ، تم تحويل مشكلة إيجاد العمق لسطح الصخور القاعدية إلى مشكلة إيجاد القيمة الأدنى لدالة هد ف محدودة في الأعماق $\psi(z)$ في صورة القياس الأول بدلاً من القياس الثاني . يمثل النموذج الرياضى تحت السطحي المفروض عددا من المستطيلات العمودية المتجانسة مغنطيسيًا ولها نفس القابلية المغنطيسية . استخدمت طريقة نيوتن المعدلة بإدماج أسلوب الفروق المحددة مع استراتيجية الفئة المتفاعلة كطريقة مهجنة لإيجاد الحد الأدنى لدالة الهدف المحدودة . تم إدماج الحدود الخطية وغير الخطية كمعلومات إضافية للحد من مشكلة تعددية الحلول الممكنة . بالإضافة إلى إمكانية حل المشكلة كمتساوية الأطراف ، تم أيضًا حلها مباشرة كمشكلة متعدية الأطراف بالقليل من التعديل . أثبتت الطريقة المقترحة بعد تطبيقها على أمثلة اصطناعية خالية من الشوشرة أو متضمنة نسبًا مختلفة ، منها أن هذه الطريقة مستقرة وقوية وتقاربية حتى للنماذج كبيرة الحجم . في النهاية ، تم تجربة الطريقة المذكورة على مثال حقلي من منطقة خليج السويس – مصر ، حيث توافقت الأعماق المحسوبة مع نتائج الآبار بالمنطقة .